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The semi-logarithmic stem number distribution and the Gini-index – structural diversity in „balanced" dbh-distributions

Semilogarithmische Stammzahlverteilungen und Gini-Index – Strukturdiversität in „Gleichgewichtsverteilungen“

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Schlüsselbegriffe: Plenterwald, ungleichaltrige Waldwirtschaft, Lorenzkurve

Abstract

Several reports show that a shift towards uneven-aged management can be observed. One of the reasons is the expectation that these forests provide a higher structural diversity, which is expected to reduce the risk of storm damage and provide a greater variety of habitats and thus increasing the biodiversity of the fauna.

Guidelines for uneven-aged management are often based on a semi-logarithmic diameter distribution, called the BDq-line, which is defined by a residual basal area (B), a maximum breast height diameter (D) and the constant rate q, describing the ratio between the stem numbers of two adjacent diameter-classes.

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Structural diversity can be characterized by the Gini-index, a measure for the inequality of the distribution of the basal area of trees. We show that this Gini-index can be derived from the parameters D and q of the BDq-line and varies between 50% and 75%. Thus, even in „balanced“ diameter distributions, the structural diversity varies considerably.

Zusammenfassung

Allgemein kann in Mitteleuropa eine Tendenz zur ungleichaltrigen Waldwirtschaft bemerkt werden. Einer der Gründe dafür ist die Erwartung, dass dies zu einer erhöhten strukturellen Diversität der Bestände führt, die ihrerseits das Windwurfrisiko vermindert und durch die Bereitstellung mehrerer Habitate zur Biodiversität der Fauna beiträgt.

Richtlinien zur Behandlung solcher ungleichaltriger Bestände beziehen sich oft auf eine semi-logarithmische Durchmesserverteilung, die jeweils durch den maximalen Brusthöhendurchmesser, D, die verbleibende Grundflächendichte, B, und eine konstante Stammzahlabnahmerate, q definiert wird.

Die Strukturdiversität von Waldbeständen wird häufig durch den Gini-Index beschrieben, ein Maß für die Ungleichverteilung der Grundflächen der Bäume des Bestandes. In dieser Arbeit wird gezeigt, dass der Gini-Index solcher semilogarithmischen Stammzahlverteilungen aus deren Parametern D und q abgeleitet werden kann und zwischen 50% und 75% liegt. So variiert also die Strukturdiversität auch in solchen Gleichgewichtsverteilungen noch beträchtlich.

1. Introduction

Huss (2014) reports for the Blackforest – and states that this may hold for the whole of Central Europe – that after a period of overusing, degrading and even devastating the forests in the 18th century, afforestation and first ideas of regular and sustainable forestry led to a common understanding of only even-aged forest management with afforestation and clearcutting as appropriate for sustainably managing forests. While until about 1800 individual tree harvesting (Plenterung) was a common method of forest use, the uncontrolled application of this method led to regulations, forbidding individual tree harvest in all public forests already in 1830. In about 1850, regular even-aged management was prescribed, which later on was replaced by regenerating old forest stands through gap cutting and by the end of the 19th century this method was again refined towards regulated individual tree harvesting (Plenterwirtschaft). During the first half of the 20th century a multitude of silvicultural systems was developed, among these the Plenter-forest system was understood as the alternative forest system, which Ammon (1937) enthusiastically described as “the only correct and true nature-near, and finally economically most advantageous silvicultural system". Nevertheless, the vast majority of Central European forests were managed in the even-aged clear cut system.

As a result of the debate on a general forest decline ("Waldsterben") in the 1980ies and subsequent catastrophic storm events by the end of the 20^{th} century, the public interest in forests increased tremendously. Given the claims of the public, different stakeholders and the politics for different forest functions, new forest management practices, like „continuous cover forestry", „nature near forestry", frequently gained ground (see e.g. Pukkala et al. 2011) or even have been prescribed by laws (see e.g. Pommerening 2002). Under these circumstances the importance of uneven-aged management grew and an increasing number of forest owners decided to abandon clear cutting and shift to individual tree harvesting, and uneven-aged management, intended to develop more stable and resilient forests with a higher degree of diversity. From this uneven-aged management a lower vulnerability against storm damage (Hahnewinkel et al. 2014), a higher diversity of habitats for birds (Diaz et al. 2005, Gruss and Schulz, 2011), small mammals (Ecke et al. 2002, Suchomel et al. 2012) and spiders (Gallé et a. 2017) have been expected.

1.1 The BDq-system

The first idea to develop a balanced distribution of breast height diameters (dbh) comes from de Liocourts (1898), who defined a reverse J-shaped dbh-distribution as stable. Meyer (1933) referred to de Liocourts, defining the balanced dbh distribution by a straight line of the logarithm of stem number over the dbh:

$ln\ (\boldsymbol{n}) = \boldsymbol{a} + \boldsymbol{k} \cdot \boldsymbol{dbh} \rightarrow \boldsymbol{n} = \boldsymbol{k}_0 \cdot \boldsymbol{e}^{-k1 \cdot dbh}$, which leads to a distribution, where $\boldsymbol{n}_i = \boldsymbol{q} \cdot \boldsymbol{n}_{i-1}$

with $\boldsymbol{n}_i$, the tree number in the $\boldsymbol{i}^{th}$ dbh-class, and $\boldsymbol{q} = \boldsymbol{e}^{k1 \cdot w}$ with **w**, the width of the dbh-class.

Although Kerr (2014) could show that de Liocourts' (1898) original dbh-distribution did not assume a constant $\boldsymbol{q}$, the idea of a balanced dbh-distribution according to Meyer's (1933) depiction was successful, and many guidelines for the management of uneven-aged forests relate to this equation, defining a certain dbh-distribution as depending on the residual basal area, B (basal area per hectare after tree-removals), the maximum dbh, D, and q, thus called BDq-system (see e.g. Cancino and Gadow 2002, Gulding 1991; Hansen and Nyland 1987, Duduman 2011). Even those who show that there are other equilibrium-distributions, at least use the semi-logarithmic dbh-distribution as a reference (e.g. O'Hara and Gersonde, 2004).

1.2 The Gini coefficient

One of the reasons for shifting management towards uneven-aged forests is the expectation that through a higher degree of structural diversity, the diversity of habitats would increase, and thus the species diversity too. Links between forest structure and biodiversity have been found in a multitude of studies (e.g. McElhinny et al. 2006; Gruss and Schulz 2011; Gallé et. al. 2017).

Among a great many of measures for the structural diversity in forests, the Gini-index (Gini, 1912) proved to very well distinguish between structures derived from different stand management systems (Zingg and Sterba 2006, Katholnig 2012).

This index is based on the Lorenz curve, which – in the case of forest stands – depicts the relative cumulative volume or basal area over the relative cumulative stem number (see e.g. Weiner and Thomas 1986, Stöcker, 2002). If all trees in a stand had the same basal area, i.e. having the same size, the Lorenz curve will be the 45°-lines in Fig. 1. The greater the inequality of the tree dimension the more does the Lorenz curve deviate from the 45°-line. The Gini coefficient is then the area between the Lorenz-curve and the 45°-line in percent of the area of the triangle below the 45°-line. Figures 1 and 2 show the Lorenz-curve and the Gini-coefficients and the stem number distributions respectively for four different stand structural types.

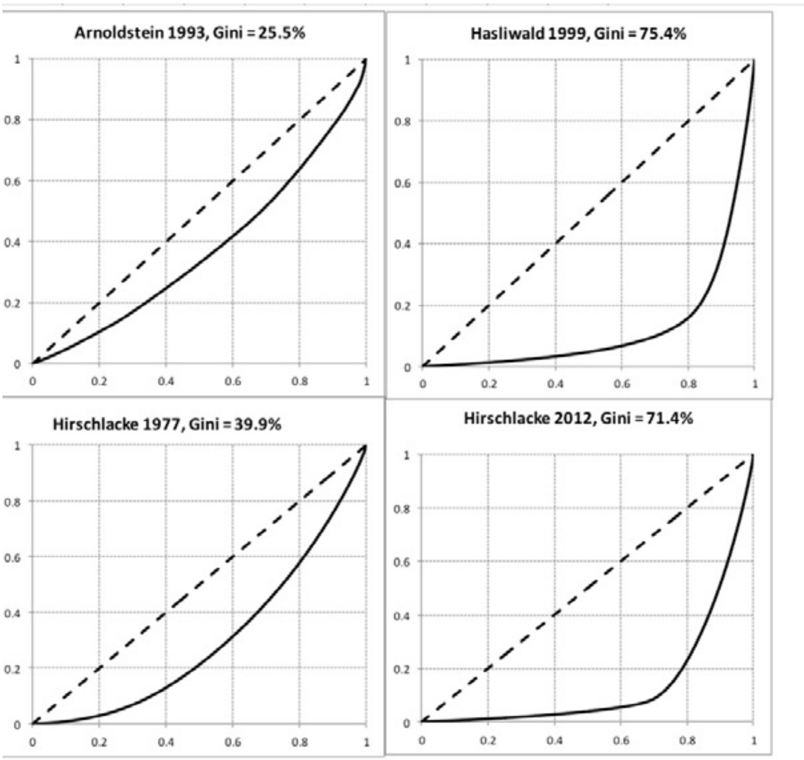

Fig.1: The Lorenz-curve for different stand types: Arnoldstein is an even-aged, young pole-stage, pure Norway spruce (Picea abies, L. KARST.) stand, not yet having been thinned actively. Hasliwald is a typical mixed White Fir (Abies alba L.)– Norway spruce (Picea abies L.) Swiss Plenterwald (see Katholnig, 2012). Hirschlacke is in 1977 a 110 year old even-aged Norway spruce stand in the Austrian part of the Bohemian Massif, which has been thinned several times after snow breakage events and only slightly thinned from below in between (Sterba, 1999). After 1977 it has been "target-diameter-harvested" according to Reininger (1987) thus, approaching an uneven-aged stand structure (Sterba & Zingg 2001) in this way.

Abb. 1: Die Lorenzkurve für verschiedene Bestandestypen: Arnoldstein ist ein junges gleichaltriges reines Fichtenstangenholz, das bisher noch nicht aktiv durchforstet wurde. Hasliwald ist ein typischer Tannen-Fichten Plenterwald. Hirschlacke ist 1977 ein 110 Jahre alter gleichaltriger Fichtenbestand im Österreichischen Teil der Böhmischen Masse, der einige Male nach Schneebruchereignissen durchforstet und dazwischen schwach niederdurchforstet worden war. Nach 1977 wurde er im Sinne von Reininger (1987) „zielstärken-genutzt" und nähert sich so einer ungleichaltrigen Bestandesstruktur.

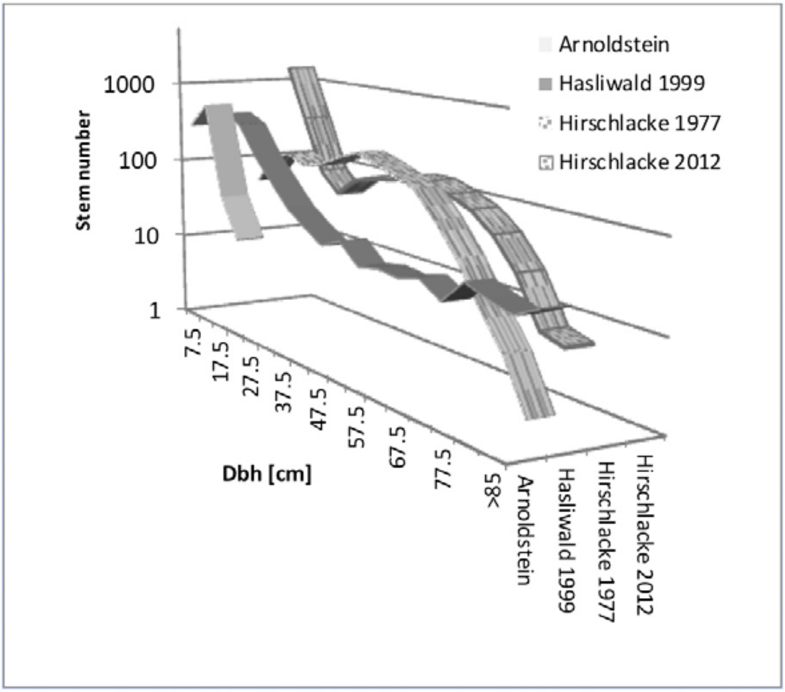

Figure 2: The dbh – distributions of the four different structural stand types. While Arnoldstein and Hirschlacke 1977 show the typical left-truncated log-normal distribution, Hasliwald and Hirschlacke 2012 show different approximations of uneven-aged, balanced dbh-distribution

Abb. 2: Die Brusthöhendurchmesser-Verteilungen der vier Bestandesstruktur-Typen. Während Arnoldstein und Hirschlacke 1977 typische links-gestutzte Log-Normalverteilungen zeigen, sind Hasliwald und Hirschlacke 2012 verschiedene Näherungen ungleichaltriger Gleichgewichtsverteilungen.

Because of this expectation of higher diversity in uneven-aged forests, the objective of this study is to show how the Gini-index can be derived from the parameters of the BDq-line, thus investigating how typically semi-logarithmic stem-number distributions vary in their Gini-coefficient, depending on the characteristics of the respective BDq-line.

2. Derivation of the Gini-Index from the BDq-line

Starting with the distribution of the stem number over the diameter at breast height (1.3 m), ***dbh***, with

$$\boldsymbol{n} = \boldsymbol{k}_0 \cdot \boldsymbol{e}^{-\boldsymbol{k}_1 \cdot \boldsymbol{dbh}} \qquad \text{Eq.(1)}$$

the cumulative stem number distribution, approximated by its integral is

$$\boldsymbol{Sn} = \int_0^{\boldsymbol{D}} \boldsymbol{k}_0 \cdot \boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{dbh}} \, d\, \boldsymbol{dbh} = \frac{\boldsymbol{k}_0}{-\boldsymbol{k}_1}\left(\boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{D}} - 1\right) \qquad \text{Eq. (2)}$$

with ***D*** the maximum diameter at breast height.

The relative cumulative stem number distribution is then

$$\boldsymbol{Sn}_{rel} = \frac{\dfrac{\boldsymbol{k}_0}{-\boldsymbol{k}_1}\left(\boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{dbh}} - 1\right)}{\dfrac{\boldsymbol{k}_0}{-\boldsymbol{k}_1}\left(\boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{D}} - 1\right)} = \frac{\boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{dbh}} - 1}{\boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{D}} - 1} \qquad \text{Eq. (3)}$$

The basal area ***ba*** is $\boldsymbol{ba} = \boldsymbol{k}_0\, \pi/4 \cdot \boldsymbol{dbh}^2 \cdot \boldsymbol{e}^{-k_1 dbh}$ and thus the cumulative basal area again approximated as integral

$$\boldsymbol{Sba} = \int_0^{\boldsymbol{D}} \boldsymbol{k}_0 \frac{\pi}{4} \boldsymbol{dbh}^2 \boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{dbh}} \, d\, \boldsymbol{dbh} = \boldsymbol{k}_0 \frac{\pi}{4}\left[\boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{D}}\left(\frac{\boldsymbol{D}^2}{-\boldsymbol{k}_1} - \frac{2\boldsymbol{D}}{\boldsymbol{k}_1^{\,2}} - \frac{2}{\boldsymbol{k}_1^{\,3}}\right) + \frac{2}{\boldsymbol{k}_1^{\,3}}\right] \qquad \text{Eq. (4)}$$

And the relative cumulative basal area is then

$$\boldsymbol{Sba}_{rel} = \frac{\boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{d}}\left(\dfrac{\boldsymbol{dbh}^2}{-\boldsymbol{k}_1} - \dfrac{2\boldsymbol{dbh}}{\boldsymbol{k}_1^{\,2}} - \dfrac{2}{\boldsymbol{k}_1^{\,3}}\right) + \dfrac{2}{\boldsymbol{k}_1^{\,3}}}{\boldsymbol{e}^{-\boldsymbol{k}_1 \boldsymbol{D}}\left(\dfrac{\boldsymbol{D}^2}{-\boldsymbol{k}_1} - \dfrac{2\boldsymbol{D}}{\boldsymbol{k}_1^{\,2}} - \dfrac{2}{\boldsymbol{k}_1^{\,3}}\right) + \dfrac{2}{\boldsymbol{k}_1^{\,3}}} \qquad \text{Eq. (5)}$$

To now be able to calculate the area under the Lorenz curve, it is necessary to find a solution to the integral

$$A_{Lorenz} = \int_0^1 Sba_{rel}\left(Sn_{rel}\right) dSn_{rel} \qquad \text{Eq. (6)}$$

By expressing ***dbh*** as a function of Sn_{rel} and substitution in the expression for ***Sba**$_{rel}$*, the above integral can be solved by substitution. Thus, the area under the Lorenz-curve is

$$A_{Lorenz} = \frac{1}{2\left(e^{-k_1 D} - 1\right)} \cdot \frac{e^{-2k_1 D}\left(k_1{}^2 D^2 + 3k_1 D + \frac{7}{2}\right) - 4e^{-k_1 D} + \frac{1}{2}}{e^{-k_1 D}\left(k_1{}^2 D^2 + 2k_1 D + 2\right) - 2} \qquad \text{Eq. (7)}$$

3. Results

Obviously the area under the Lorenz-curve, and the Gini-index resulting from it, can be explicitly expressed as a function of the maximum diameter, ***D*** and the slope $\boldsymbol{k_1}$ of the BDq-line.

The basal area, i.e. the stand density, does not influence the Gini-coefficient. As can be seen from Figure 3, and as having been expected, the Gini-index and thus the structural diversity of a “balanced dbh-distribution” increases with increasing slope of the BDq-line and with increasing maximum dbh.

Note that for $\boldsymbol{q=1}$, $\boldsymbol{k_1=0}$, and a uniform stem number distribution, the resulting Gini-Index is 50%, and for $\boldsymbol{q=\infty}$, $\boldsymbol{k_1=-\infty}$, the Gini-index approaches 75%. Thus for any ***q*** , the Gini-index is limited between 50% and 75%.

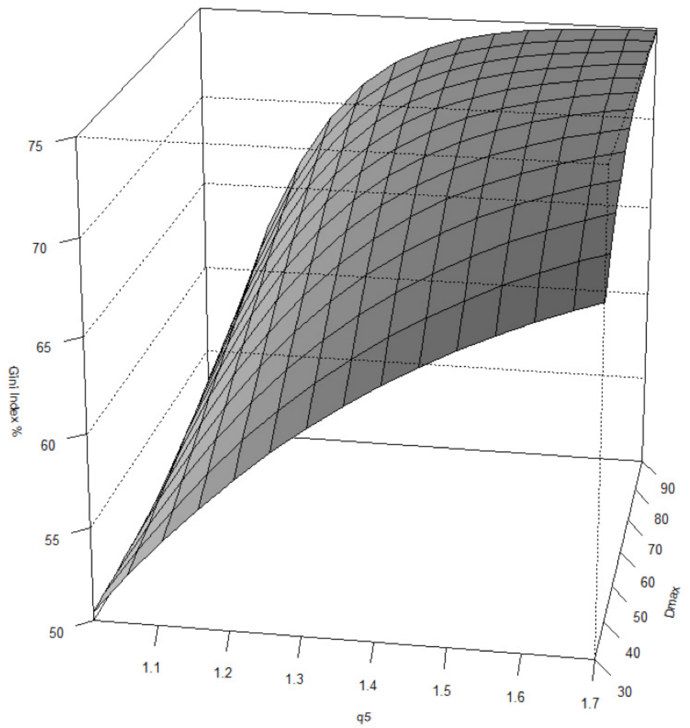

Figure 3: The Gini-index for different BDq-lines, depending on the maximum dbh, D_{max} and the slope of the BDq-line. Note that, in this figure, the coefficient k_1 results in q_5 by solving the equation $\mathbf{q} = \mathbf{e}^{k_1 \cdot w}$ for $\mathbf{k}_1$ for a dbh-class width of 5 cm resulting in $\mathbf{k}_1 = \ln \mathbf{q}_5 /5$.

Abbildung 3: Der Gini-Index für verschiedene semilogarithmische Durchmesserverteilungen in Abhängigkeit vom maximalen BHD, D_{max}, und dem Gefälle der Verteilungsfunktion, k_1. In dieser Abbildung wird der Koeffizient k_1 aus q_5 hergeleitet, indem die Gleichung $\mathbf{q} = \mathbf{e}^{k_1 \cdot w}$ für eine Klassenbreite von 5 cm als $\mathbf{k}_1 = \ln \mathbf{q}_5 /5$ gelöst wird.

3.1 Example

For demonstrating the findings above, we used the data of the Hirschlacke-stand in more detail. From a targeted maximum dbh of 80 cm, and a targeted residual basal area of 50 m^2/ha results a q_5 of 1.28 for a targeted balanced dbh-distribution. The resulting Gini-index according to equation (7) is 70.7%. Figure 4 shows how the Gini-index of the Hirschlacke stand evolves through 35 years of target diameter management in comparison to the aim of a balanced dbh-distribution with the targeted maximum dbh and residual basal area.

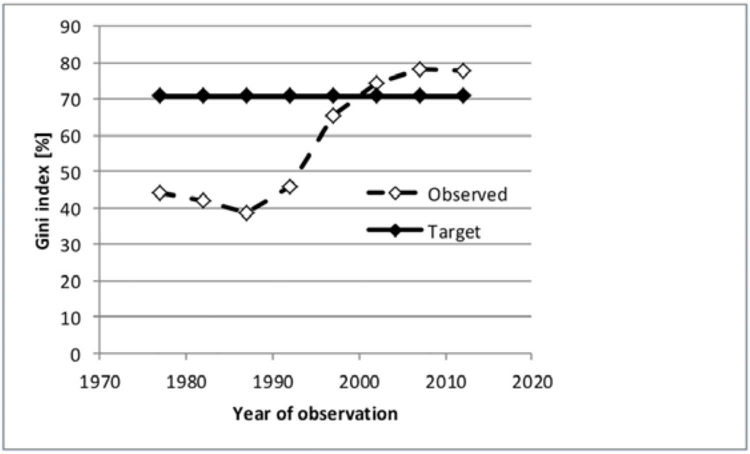

Figure 4: The development of the Gini-index in the Hirschlacke-stand compared to the targeted balanced dbh-distribution with a maximum dbh of 80 cm and a residual basal area of 50 m²/ha

Abb. 4: Die Entwicklung des Gini-Index in der „Hirschlacke" im Vergleich zu einer Gleichgewichtsverteilung mit einem maximalen Brusthöhendurchmesser von 80 cm und einer verbleibenden Grundflächendichte von 50 m²/ha

The development shows until 1987 a Gini-index below 50%, which is typical for even-aged stands. Afterwards the Gini-index increases fast and is larger than 75% since 2007. In 2012 the stand is still slightly more structural diverse than the targeted balanced dbh-distribution. This shows that when targeting the balanced dbh-distribution, the structural diversity will have to decrease a little bit.

4. Discussion

Theoretically the Gini-indices of BDq-lines vary between 50% and 75%. The empirical studies of reversed J-shaped dbh-distributions in uneven-aged and Plenter-forests report average Gini-indices of about 63% (Katholnig 2012, Sterba & Zingg 2006), which is just in the middle of the theoretical range. The 57 stands investigated by Sterba & Zingg (2006) included typical Plenter-forests on the one side, and regularly thinned even-aged monospecific and mixed stands and coppice forests on the other side. The average Gini-index of these other investigated stand types was 44% and significantly lower than that of the 20 Plenter-forests, and as well below the theoretical minimum of the Gini-indices of perfect BDq-lines. Please note that a Gini-index below 50% is to be expected in even-aged stands, because they do not have uniform *dbh*-distributions (which would have a Gini-index of 50%), but rather truncated log-normal dbh-distributions.

When fitting the *dbh*-distribution by a semi-logarithmic regression, Katholnig (2012) followed Zingg and Duc (1998) using the variance around this line as a measure of deviation from the perfect BDq-line. In the uneven-aged forest, the Gini-index decreased with this deviation from the perfect BDq-line, explaining 22% of the variation of the Gini-index. Considering the regression between Gini-index and the deviation from the BDq-line as found by Katholnig (2012), the Gini-index for a deviation of zero results in a Gini-index of 71% which is quite near its theoretical maximum.

An example shows how the Gini-index, as developed from a targeted semi-logarithmic dbh-distribution can serve as a reference for the result of the management of a given stand.

5. Conclusions

From a multitude of observed stem number distributions in uneven-aged forests and Plenter-forests, it can be seen that the average Gini-index of such forests is about 60% and significantly higher than in forest stands managed according to even-aged management strategies.

Not only deviations from a perfect BDq-line affect the Gini-index. Even for balanced dbh-distributions described by semi-logarithmic equations, the BDq-line, the Gini-coefficient can be explicitly described by the maximum dbh and the slope of the BDq-line. This theoretical Gini-index of the BDq-lines varies between 50% and 75% and increases with increasing maximum dbh and increasing slope of the BDq-line.

The Gini-index of a targeted semi-logarithmic dbh-distribution, as developed from Equation (7) can serve as a reference for the observed development of the structural diversity as it results from the management of a stand.

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