Nonlinear height-diameter models for Hungarian oak (*Quercus frainetto* Ten.) in Dumanlı Forest Planning Unit, Çanakkale/Turkey

Nichtlineare Höhen-Durchmesser-Modelle für ungarische Eiche (*Quercus frainetto* Ten.) im Forstbetrieb Dumanlı, Çanakkale/Türkei

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Schlüsselbegriffe:	Allometrie, Baumdimensionen, Autokorrelation, Heteroskedastizität, Forstwirtschaft, Waldinventur

Abstract

The measurement of tree heights is a tedious task and highly time intensive. Tree heights are closely linked to tree diameters that are measured simply. Therefore, total tree height is commonly predicted as a function of its diameter. In this study, two-, three-, and four-parameter height-diameter (H-D) models including only tree diameter as the independent variable were developed using a total of 470 observations. The results showed that the models were accurate with good fit indices. The modified von Bertalanffy model had the best predictive performance with an R_{adj}^2 of 0.80. The characteristics of the predictions were found to be consistent with biological expectations. The residuals by the models were distributed normally and independently. Consequently, the nonlinear H-D models developed can reliably be used for predicting the heights of Hungarian oak (*Quercus frainetto* Ten.) trees in Dumanlı Forest Planning Unit, Çanakkale/Turkey. In addition, height-diameter relations may be influenced by stand conditions. Therefore, the nonlinear H-D models developed may have a limitation when they will be used in the outside of study area.

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Zusammenfassung

Direkte Baumhöhenmessungen im Feld sind kostspielig und zeitaufwendig. Baum-
höhe korreliert eng mit Stammdurchmessern, die leichter zu messen sind. Daher wird
Baumhöhe häufig als Funktion des Stammdurchmessers geschätzt. In dieser Studie
entwickeln wir zwei-, drei- und vierparametrige Höhen-Durchmesser-Modelle mit-
hilfe von 470 beobachteten Bäumen. Unsere Ergebnisse zeigen, dass alle Modelle mit
guter Anpassung an die Beobachtungen akkurat sind. Das modifizierte Bertalanffy-
Modell hat die beste Performance mit einem R_{adj}^2 von 0.80. Die Muster der Schätz-
funktionen waren konsistent mit biologischen Erwartungen. Die Residuen waren
normalverteilt und unabhängig. Die entwickelten Höhen-Durchmesser-Modelle er-
scheinen zuverlässig für die Schätzung von Baumhöhen bei Ungarischer Eiche im
Forstbetrieb Dumanlı, Çanakkale/Türkei. Höhen-Durchmesser-Beziehungen werden
durch Bestandesbedingungen beeinflusst. Daher ist eine Anwendung der hier ent-
wickelten nichtlinearen Modelle außerhalb des Untersuchungsgebietes kritisch zu
beurteilen.

1 Introduction

Tree height is a key variable in modeling forest attributes such as individual tree volu-
me (Subedi *et al.*, 2021), tree biomass (Ledermann and Gschwantner, 2006), and tree
carbon content (Thurnher *et al.*, 2013). Inclusion of tree height to the individual-tree
and the whole-stand models improves their accuracy and adaptability. In addition,
an intense field sampling effort is needed for accurately measuring tree heights. The-
refore, in general, the functional allometric relationships between tree diameters and
tree heights are established for predicting tree heights that are not measured in the
forest inventory study. Because the measurement of tree diameter is relatively easy,
and it can be obtained with a high degree of accuracy (Yang and Burkhart, 2020).

The H-D models have been developed for various tree species in Turkey, for instance,
by Sönmez (2009) for Oriental spruce (*Picea orientalis* L.), by Mısır (2010) for Anato-
lia trembling aspen (*Populus tremula* L.), by Diamantopoulou and Özçelik (2012) for
Taurus cedar (*Cedrus libani* A. Rich.), Cilicica fir (*Abies cilicica* Carr.) and Brutian pine
(*Pinus brutia* Ten.), by Özçelik *et al.* (2013) for Crimean juniper (*Juniperus excels* Bieb),
by Özçelik *et al.* (2014) for Brutian pine, Black pine (*Pinus nigra* Arnold) and Taurus
cedar, by Ercanlı (2015) for Oriental beech (*Fagus orientalis* Lipsky), by Özdemir and
Saraçoğlu (2016) for Oak stands (*i.e.*, Sessile oak [*Quercus petraea* [Matt.] Liebl.], Hun-
garian oak [*Quercus frainetto* Ten.] and Turkey oak [*Quercus cerris* L.]), by Karatepe *et
al.* (2022) for Taurus cedar plantation and by Seki and Sakıcı (2022) for Black pine. In
Europe, America and Asia, H-D models have been developed by Lynch *et al.* (2005) for
Cherrybark Oak (*Quercus pagoda* Raf.) in southeastern of United States, by Adame *et
al.* (2008) for Pyrenean oak (*Quercus pyrenaica* Willd.) in northwest of Spain, by Paulo

et al. (2011) for Cork oak (*Quercus suber* L.) in Portugal, by Koulelis *et al.* (2018) for Hungarian oak, European beech (*Fagus sylvatica* L.) and Bulgarian fir (*Abies borissi-regis* Mattf.) in Greece, by Ogana and Gorgoso-Varela (2020) for Maritime pine (*Pinus pinaster* Ait) in northwest of Spain, by Vanderschaaf (2020) for Loblolly pine (*Pinus tae-da* L.), Longleaf pine (*Pinus palustris* P. Mill.), Shortleaf pine (*Pinus echinata* Mill.) and Slash pine (*Pinus elliottii* Engelm.) in Louisiana, Mississippi, and Texas in United States, by Raptis *et al.* (2021) for Black pine in Greece and by Wu *et al.* (2022) for various tree species (*e.g.*, Brown oak [*Quercus semecarpifolia* Sm.] and Blue Japanese oak [*Quercus glauca* Thunb.]) in Sichuan Province, China, by Skudnik and Jevšenak (2022) for various tree species (*e.g.*, European beech, Norway spruce [*Picea abies* Karst], Sessile oak and Turkey oak) in Slovenia.

The literature suggests that the nonlinear models are flexible, interpretable, and robust (Archontoulis and Miguez, 2015), and thus they are commonly used in modeling tree heights. Huang *et al.* (1992) developed linear and nonlinear H-D models for predicting various tree species' heights. Huang *et al.* (2000) established the nonlinear H-D models for White spruce (*Picea glauca* [Moench] Voss) in different ecoregions. Similarly, Seki and Sakici (2022) built the generalized nonlinear H-D models by considering the ecoregion effects for Crimean pine (*Pinus nigra* subsp. *pallasiana* [Lamb.] Holmboe). Ogana and Ercanli (2022) developed a generalized nonlinear H-D model derived from the Meyer model using a large dataset composed by a total of 1,736 observations from many tree species. The nonlinear models have some advantages, providing a trajectory exhibiting positive slope across the range of the observed data, inflection point and asymptote. Therefore, they can competently provide the predictions that are compatible with the biological realism expressed in the study of Yuancai and Parresol (2001). For instance, Adame *et al.* (2008) evidenced the sigmoidal pattern for height-diameter relationships when they used the nonlinear model having two parameters. Ng'andwe *et al.* (2019) achieved the predictions that are in agreement with the *S*-shaped pattern using the logistic-type nonlinear H-D model.

In Turkey, the forest lands have been managed considering multiple ecosystem services since the last twenty-two years (Zengin *et al.*, 2013). Furthermore, according to the forestry statistics (URL-1, 2020), the amount of wood-based forest products increases more every year in Turkey due to more forest lands under management. Therefore, it is necessary to develop forest growth and yield models for sustainable management of the forest lands. H-D models are important for accurate predictions of forest yield and growth (Özçelik *et al.*, 2018; Temesgen *et al.*, 2007).

There are eighteen subspecies of Oak in Turkey and they occupy 29% (approximately 6.7 million hectares) of the total forest land in Turkey (URL-1, 2020). However, there is very little information on their height-diameter relationships. According to our knowledge, a H-D model is not yet available for Hungarian oak stands in Dumanlı Forest Planning Unit in Çanakkale region. The objective of this study was to evaluate nonlinear H-D models by using the goodness-of-fit statistics and by analyzing the

characteristics of the residuals and hence to identify the most appropriate model for Hungarian oak stands in the study area.

2 Material

2.1 Study Area

This study was conducted in the Dumanlı Forest Planning Unit located within the borders of the Lapseki State Forest Enterprise, Çanakkale Regional Directorate of Forestry (26° 41′ 42″ – 26° 56′ 40″ E. 40° 10′ 22″ – 40° 18′ 14″ N., WGS84 Zone 35N) in Turkey (Figure 1). The elevation of the study area varied from between 50 and 723 m with an average slope of 22%. While the highest and the lowest temperatures observed in the study area were 39 °C and -11.2 °C, the annual average precipitation was approximately 630.7 mm. The Walter and Lieth climate diagram was prepared to show the rainfall distribution and the temperature variation in the study area (Figure 2). The total area of the study site is 19,564.4 ha, and approximately 69% of that site is covered with the forests where Brutian pine, Crimean pine and Hungarian oak are widely distributed. Hungarian oak stands occupy approximately 16% (3130.3 ha) of the total forest area.
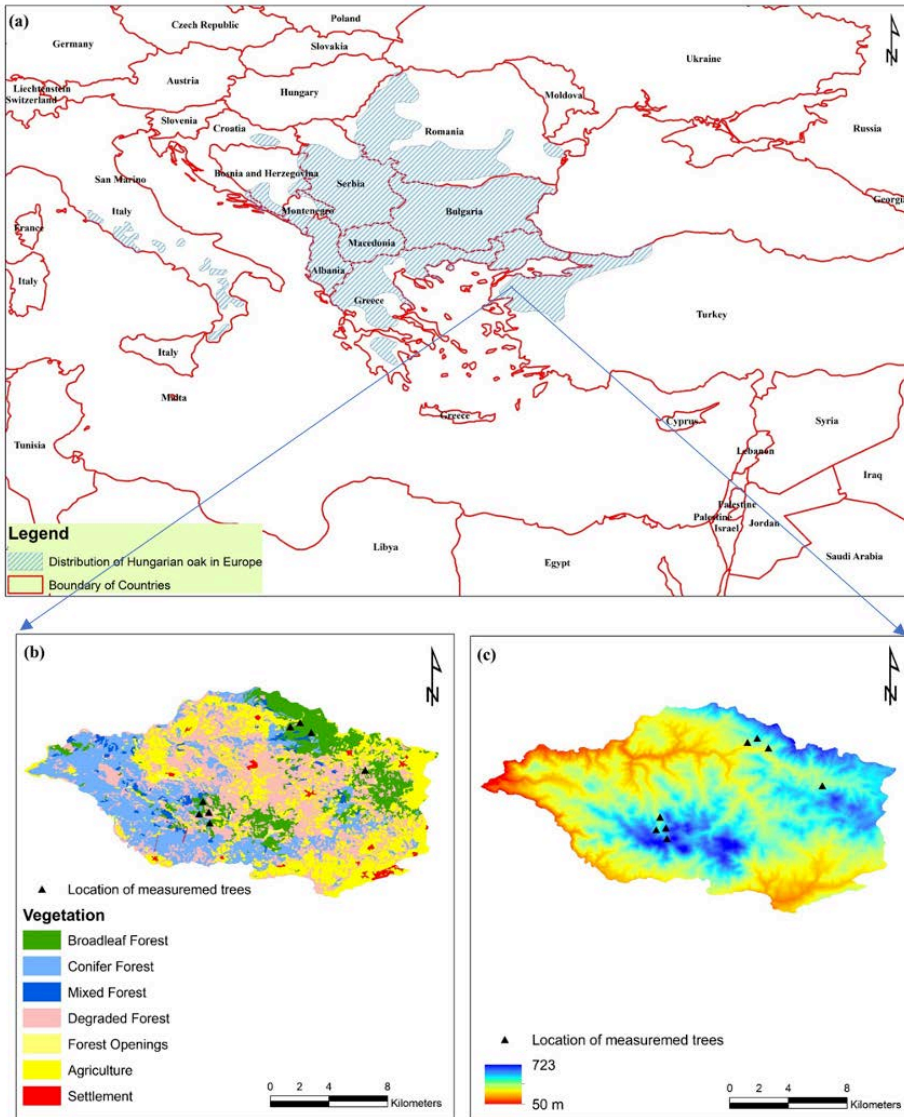
Figure 1: Distribution of Hungarian oak in Europe (a), and the conditions of study area in terms of vegetation (b) and elevation (c).

Abbildung 1: Verteilung der Ungarischen Eiche in Europa (a) und die Position der beobachteten Bäume in der Untersuchungsregion hinsichtlich Vegetationsklassen (b) und Seehöhe (c).
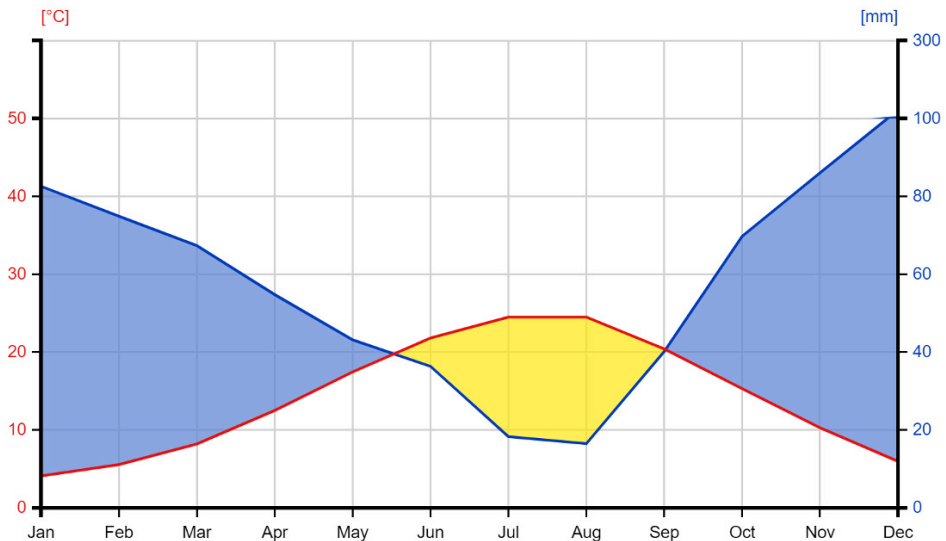
Figure 2: The distributions of temperature and rainfall in the study area (Zepner et al., 2021). The blue and red lines inside the figure show the curves of precipitation (the right axis) and temperature (the left axis) for the period of 1990-2019, respectively. The colors "yellow and "blue" between the curves suggest the months under arid and humid climate conditions, respectively.

Abbildung 2: Lufttemperatur und Niederschlag in der Untersuchungsregion (Zepner *et al.*, 2021). Die dunkelblaue und die rote Linie zeigen die Verteilung von Niederschlag (rechte Achse) und Temperatur (linke Achse) für den Zeitraum 1990–2019. Die gelben und hellblauen Flächen zwischen den Linien zeigen Monate mit ariden und mit humiden Klimabedingungen.

2.2 Sampling and data collection methods

Diameter and height measurements were taken from a total of 470 trees in even-aged monospecific Hungarian oak stands originated from seed (Table 1). The sample trees were obtained from the stands to represent the available range of developmental ages in the studied area. Figure 2 shows a few examples for a very young forest <8 cm (a), a young forest 8-19.9 cm (b), a mature forest 20-35.9 cm (c) and an over mature forest 36-51.9 cm (d). The breast circumference of trees at a height of 1.30 m and their total heights were measured using with 1% precision using the Vertex IV ultrasound instrument. These measurements were carried out in April 2022. Some photographs from monospecific Hungarian oak stands sampled were presented in Figure 3.

Figure 3: Photos from even-aged monospecific Hungarian oak stands; a) very young b) young c) mature d) over mature.

Abbildung 3: Aufnahmen der untersuchten gleichaltrigen Bestände, dominiert durch Ungarische Eiche; sehr jung (a), jung (b), reif (c) und überreif (d).

Table 1: Descriptive statistics of tree height and tree diameter measured in this study.

Tabelle 1: Beschreibende Statistiken der Baumhöhen und Stammdurchmesser, verwendet in dieser Studie.

Measures	Model data (n=377)		Test data (n=93)	
	Height (m)	Diameter (cm)	Height (m)	Diameter (cm)
Minimum	3.2	4.3	4.4	4.0
Maximum	26.5	51.4	26.0	49.3
Mean	14.0	22.2	13.5	21.7
Standard Deviation	5.2	10.4	5.3	10.2

2.3 Method

In this study, two, three and four-parameter nonlinear models were fitted for Hungarian oak stands. General expressions of them were given below. Eqs (1)-(6) models were provided from the study of Fang and Bailey (1998). Eq. (7) is a modified von Bertalanffy model (Korkmaz and Üçkardeş, 2013). During the development of models, the starting values of the regression parameters were specified using the GenSA package in R environment (R Core Team, 2022; Xiang *et al.*, 2013). The significance of the estimated parameters was tested calculating *p*-values after each model was performed. It is expected that p value should be lower than 0.05, showing that the estimated parameter is statistically significant.

$$H = 1.3 + b_1 D^{b_2} + e \tag{1}$$

$$H = 1.3 + b_1 e^{\left(\frac{b_2}{D}\right)} + e \tag{2}$$

$$H = 1.3 + b_1 D^{\left(b_2 D^{-b_3}\right)} + e \tag{3}$$

$$H = 1.3 + \frac{b_1}{\left(1 + b_2 e^{(-b_3 D)}\right)} + e \tag{4}$$

$$H = 1.3 + b_1 \left(1 - e^{\left(-b_2 D^{b_3}\right)}\right) + e \tag{5}$$

$$H = 1.3 + b_1 \left(1 - e^{(-b_2 D)}\right)^{b_3} + e \tag{6}$$

$$H = 1.3 + b_1 + (b_2 - b_1)\left(1 - e^{(-b_3 D)}\right)^{1/b_4} + e \tag{7}$$

where H is individual tree height (m), D is diameter at breast height (cm), $b_{1\text{-}4}$ is regression coefficient to be estimated, e is residual.

Statistical evaluation of the models considered was made using common goodness-of-fit measures showing the accuracy and the adaptability of them (Eq. (8) to (12)). A ranking method developed by Poudel and Cao (2013) was used for determining the best model.

$$R^2_{adj} = 1 - \left(1 - \frac{\sum_{i=1}^n (H - \widehat{H})^2}{\sum_{i=1}^n (H - \overline{H})^2}\right)\left(\frac{n-1}{n+p-1}\right) \tag{8}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (H - \widehat{H})^2}{n}} \tag{9}$$

$$MAD = \frac{\sum_{i=1}^n |H - \widehat{H}|}{n} \tag{10}$$

$$AIC = -2 \times (L) + 2 \times p \tag{11}$$

$$BIC = -2 \times log(L) + log(n) \tag{12}$$

where H and $\hat{H}$ are the observed and predicted height values, respectively, $\bar{H}$ is mean of the observed values, n and p are the number of observations and parameters, respectively, L is the maximized loglikelihood value for the fitted models, R_{adj}^2 is the adjusted coefficient of determination, RMSE is root mean squared error, MAD is mean absolute bias, AIC is Akaike information criterion (Akaike, 1998), and BIC is Bayesian information criterion (Schwarz, 1978).

The overall performance of the models was tested using the model data. In addition, the models were also examined using a test data. Test statistic scores were calculated based on only RMSE.

The statistical assumptions were checked by analyzing the residual distributions of the models. In a regression model, the residual distributions should be distributed normally and have a constant variance. In addition, they should be tested using Durbin-Watson statistic (DW) for the presence of possible autocorrelation (Chatterjee and Simonoff, 2013; Zuur *et al.*, 2010). The normality test of the residuals should be made using a proper test. In this study, the Kolmogorov-Smirnov test was used.

3 Results

The estimated parameters of the nonlinear H-D models were statistically significant at 5% level. Table 2 shows their values with the standard errors.

Table 2: Parameter estimates with standard error in brackets for nonlinear H-D models developed for Hungarian oak stands. The parameters and their standard errors were estimated using entire dataset.

Tabelle 2: Schätzparameter mit Standardfehler in Klammer für nichtlineare H-D-Modelle, entwickelt für ungarische Eichenbestände. Die Parameter wurden mit dem gesamten Datensatz ermittelt.

Equations	b_1	b_2	b_3	b_4
Eq. (1)	1.8986 (0.1207)	0.6525 (0.0193)		
Eq. (2)	28.029 (0.5560)	-13.474 (0.4397)		
Eq. (3)	36.5 (3.7877)	-7.6419 (0.9545)	1.056 (0.0776)	
Eq. (4)	21.193 (0.42636)	6.2322 (0.5202)	0.1230 (0.0069)	
Eq. (5)	22.598 (0.9161)	0.0223 (0.0034)	1.2578 (0.0709)	
Eq. (6)	23.735 (1.049)	0.0560 (0.0077)	1.346 (0.1362)	
Eq. (7)	6.0688 (0.4844)	20.924 (0.4779)	0.1212 (0.0133)	0.1444 (0.0382)

The results showed that Eq. (7) having the smallest ranking score (5.1) had the best fit. Eq. (1) having the greatest ranking score (35.0) had the worst fit (Table 3). The error statistics indicated that all models developed had an acceptable accuracy for predicting total tree heights. In addition, the relative ranking scores implied that the nonlinear H-D models developed could be divided into subgroups based on their performance. The first group, which was the most successful, was comprised of Eq. (4) and Eq. (7). Second group, which had moderate success, was consisted of Eq. (3), Eq. (5), and Eq. (6). Third group, which had the worst predictive performance, was composed of Eq. (1) and Eq. (2).

Table 3: The goodness-of-fit statistics and relative ranking scores (after Poudel and Cao 2013) of nonlinear H-D models developed for Hungarian oak stands, using model data.

Tabelle 3: Statistiken zu Anpassungsgüte und Relative Ranking Scores (nach Poudel and Cao 2013) für die getesteten nichtlinearen H-D-Modelle für ungarische Eiche unter Verwendung aller Beobachtungen.

Equations	R_{adj}^2	RMSE	MAD	AIC	BIC	Relative ranking scores[†]
Eq. (1)	0.7605	2.5413	1.9522	1775.1	1783.0	35.0
Eq. (2)	0.7728	2.4752	1.9466	1755.2	1763.1	28.5
Eq. (3)	0.7855	2.4053	1.8562	1734.6	1746.4	19.1
Eq. (4)	0.8043	2.2972	1.7866	1700.0	1711.7	6.2
Eq. (5)	0.7928	2.3640	1.8411	1721.6	1733.4	14.6
Eq. (6)	0.7908	2.3751	1.8458	1725.1	1736.9	15.8
Eq. (7)	0.8065	2.2844	1.7763	1696.7	1712.5	5.1

The results based on test data showed that RMSE values ranged from 2.6389 to 2.7775. According to test performance of the nonlinear H-D models, Eq. (4) had the best fitness with an RMSE of 2.6389 m followed by Eq. (7) with an RMSE of 2.6447 m. The RMSE values for Eq. (1), Eq. (2), Eq. (3), Eq. (5), and Eq. (6) were 2.7070 m, 2.7775 m, 2.6664 m, 2.6765 m, and 2.6732 m, respectively.

Model behaviors against actual data were presented in Figure 4. As shown in the figure, all models behaved similarly, and they described well the patterns in observed height-diameter pairs. Goodness-of-fit statistics showed that Eq. (7) is the best with the highest R_{adj}^2 and the smallest RMSE, AIC and BIC (Fig. 5). As the number of the parameters increased, the success of the models improved. However, this improvement in models' success was not significant enough to be considered (a 0.3 m decrease in RMSE).
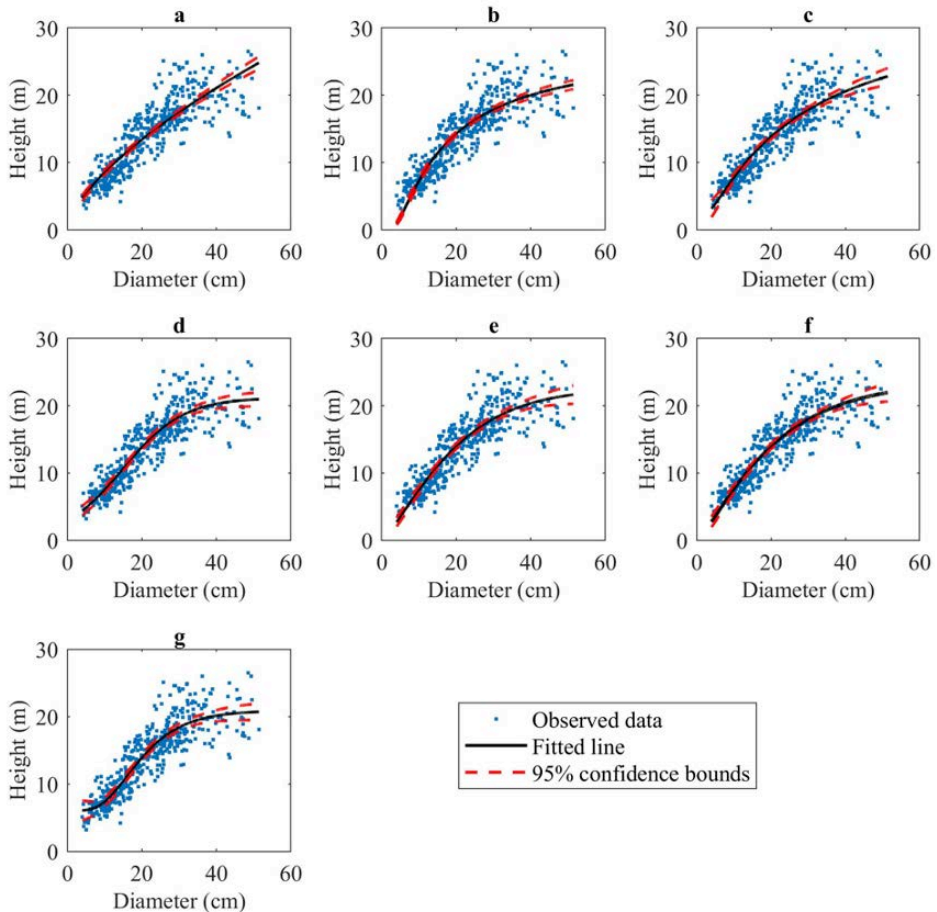
Figure 4: Plots of nonlinear H-D models with fitted lines and confidence bounds at 95% level (a, b, c, d, e, f, and g graphs represent Eq. (1), Eq. (2), Eq. (3), Eq. (4), Eq. (5), Eq. (6) and Eq. (7), respectively).

Abbildung 4: Plots der nichtlinearen H-D-Modelle mit Beobachtungen und Konfidenzintervallen auf 95%-Niveau (Panele a, b, c, d, e, f und g zeigen Gleichung (1), Gl. (2), Gl. (3), Gl. (4), Gl. (5), Gl. (6) und Gl. (7)).

Fig. 5 indicated that there was no autocorrelation among the residuals of the nonlinear H-D models except for Eq. (1). Durbin-Watson statistic for Eq. (1) was 1.8095 with a *p*-value of 0.0385, which suggested that the residuals were correlated in the Eq. (1). In addition, Fig. 6 implied that all models except for Eq. (1) provided nearly the constant variance across the range of the data. Eq. (1) resulted in dispersed residual variance. All models except for Eq. (4) and Eq. (7) overestimated the heights of shorter trees. But that failure did not actually seem like a remarkable problem owing to the

small number of the measurements for smaller trees (it accounted for approximately 4% of all data). In addition, it was observed that the residuals tended to disperse slightly following the increased fitted values. However, this dispersion was not evident enough to prove the existence of possible heteroscedasticity problem.
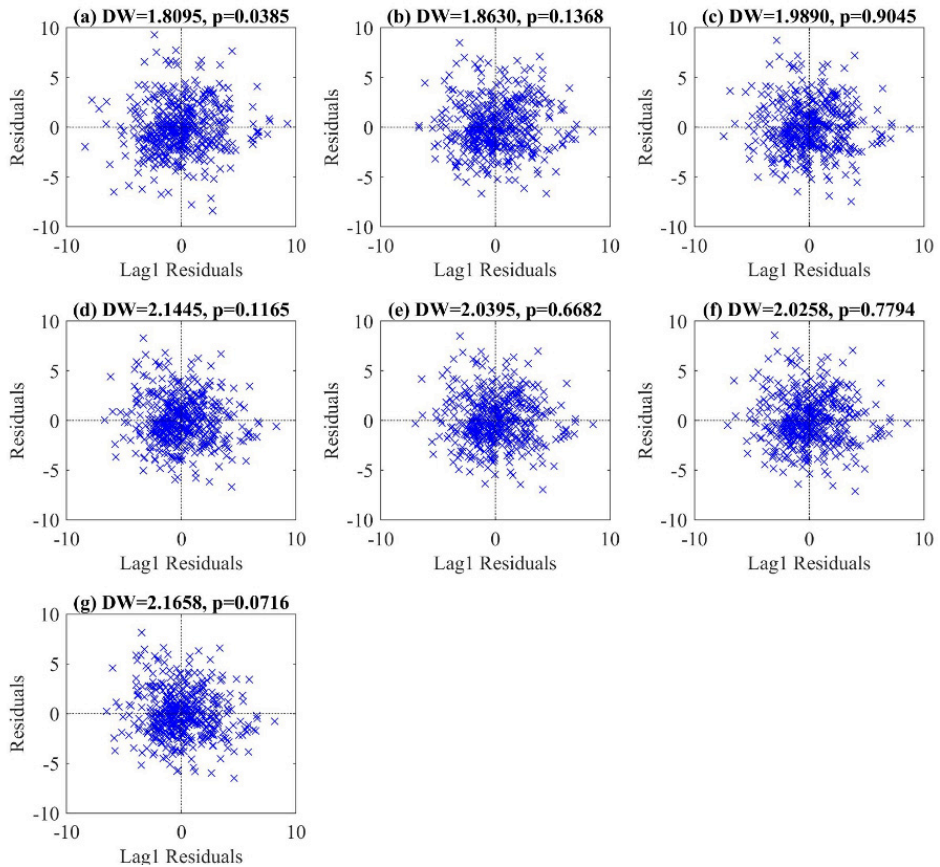

Figure 5: Plots of residual versus Lag1 residuals (a, b, c, d, e, f, and g graphs represent Eq. (1), Eq. (2), Eq. (3), Eq. (4), Eq. (5), Eq. (6) and Eq. (7), respectively).

Abbildung 5: Plots der Residuen versus Lag1 Residuen (Panele a, b, c, d, e, f und g zeigen Gleichung (1), Gl. (2), Gl. (3), Gl. (4), Gl. (5), Gl. (6) und Gl. (7)).

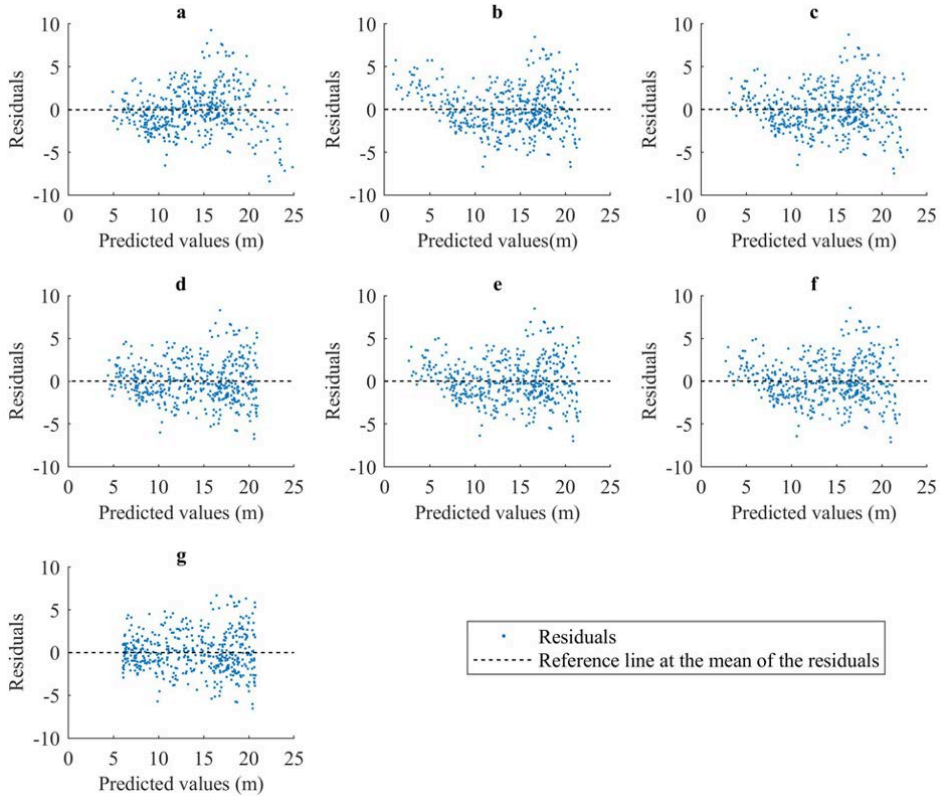Ferhat Bolat, Okan Ürker, Alkan Günlü

Figure 6: Scatter plots for the residuals of nonlinear H-D models versus fitted values (a, b, c, d, e, f, and g graphs represent Eq. (1), Eq. (2), Eq. (3), Eq. (4), Eq. (5), Eq. (6) and Eq. (7), respectively).

Abbildung 6: Streuplots der Residuen der nichtlinearen H-D-Modelle gegenüber den Beobachtungen (Panele a, b, c, d, e, f und g zeigen Gleichung (1), Gl. (2), Gl. (3), Gl. (4), Gl. (5), Gl. (6) und Gl. (7)).

Fig. 7 indicated that Eq. (4) and Eq. (7) showed more robust behavior across the diameter classes, especially in diameter classes including the small number of samples. Ultimately, both the goodness-of-fit statistics and the residual behaviors showed that Eq. (4) and Eq. (7) were more successful than their counterparts, and they met fully the expected statistical assumptions.
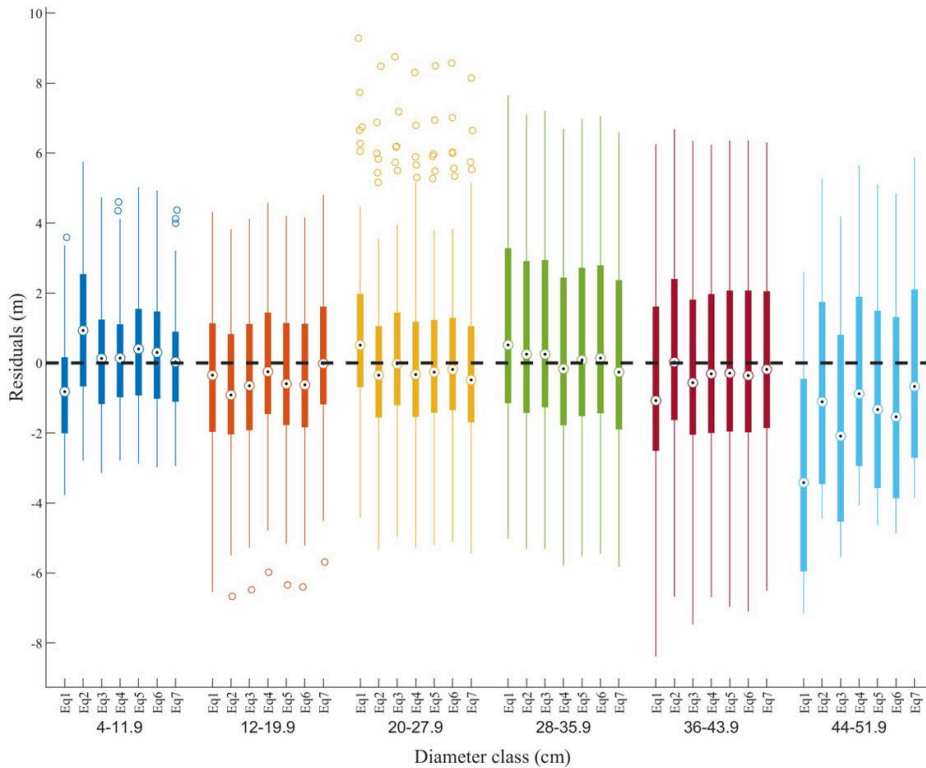
Figure 7: The residuals boxplots categorized by diameter classes for nonlinear H-D models. Eqs (1) to (7) are ordered from left to right in each class. In each boxplot, the bottom and the top of the box are 25th and 75th percentile of the residual, respectively. The central mark inside the box shows the median of the residual. The 'o' marker outside the box indicates the outliers.

Abbildung 7: Boxplots der Residuen gruppiert nach Durchmesserklassen für die nicht-linearen H-D Modelle. Gleichungen (1) bis (7) sind von links nach rechts geordnet in jeder Klasse. In jedem Boxplot zeigt die Box die 25% und 75% Quartile an. Der Punkt in der Mitte der Box ist der Median. Die „o" Markierungen außerhalb der Box sind Ausreißer.

Kolmogorov-Simirnov test showed that the residual distributions of Eq. (1)-(7) were distributed normally (p>0.05). Strong similarities were observed between the cumulative distribution of the residuals and that of the standard normal distribution, which confirmed the results of the normality test (Fig. 8).
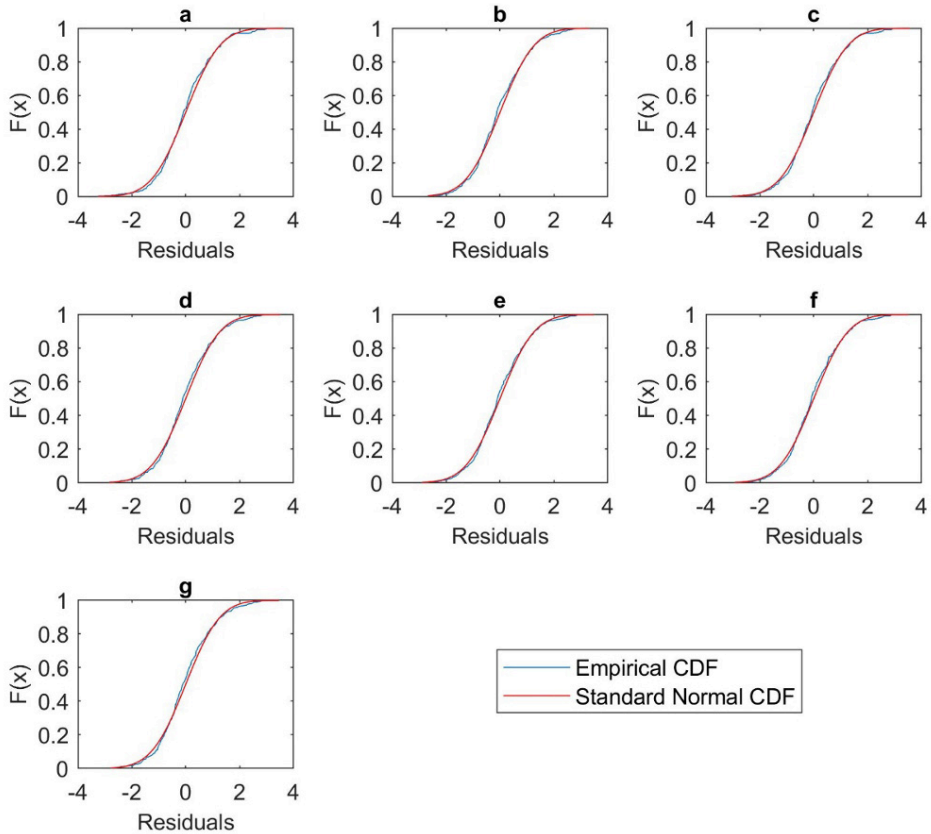
Figure 8: Line plots associated with comparing the evidenced cumulative residual distribution to the expected cumulative residual distribution (a, b, c, d, e, f, and g graphs represent Eq. (1), Eq. (2), Eq. (3), Eq. (4), Eq. (5), Eq. (6) and Eq. (7), respectively).

Abbildung 8: Linienplots zum Vergleich der tatsächlichen kumulierten Residuenverteilung mit der erwarteten kumulierten Residuenverteilung (Panele a, b, c, d, e, f und g zeigen Gleichung (1), Gl. (2), Gl. (3), Gl. (4), Gl. (5), Gl. (6) und Gl. (7)).

4 Discussion

Height-diameter models provide valuable information on economic value of a standing timber (Goulding and Murray, 1976) and timber production capacity of a stand (Newton *et al.*, 2005) as well as stand structure (Staudhammer and LeMay, 2001; Vacek *et al.*, 2014) and growth and yield (Montero *et al.*, 2001; Vospernik and Eckmuellner, 2012). In addition, the H-D models are important for the remote sensing studies as well (Anwar *et al.*, 2018; Jucker *et al.*, 2017). While in remote sensing studies tree

height can be obtained using remote sensing sensors having high spatial resolution such as IKONOS, QuickBird and Spot-5, tree diameter cannot directly be measured from them. With this regard, the inverse of the H-D models can be very useful for obtaining tree diameter (Hirata, 2008; Hirata *et al.*, 2014, 2009; Salas *et al.*, 2010). On the other hand, recently the remote sensing sensors having a great spatial (*e.g.*, light detection and ranging, LiDAR) enable to extract the 3-D structure of individual trees and forests (*i.e.*, height, crown height and crown diameter) (Kwak *et al.*, 2007). The accuracy of these estimations can be examined using H-D models (Donev *et al.*, 2021).

Goodness-of-fit statistics showed that the nonlinear H-D models developed were adequately fitted to the observed data. But, Eq. (1), Eq. (2), Eq. (3), Eq. (5) and Eq. (6) produced the biased predictions for both smaller and bigger trees. Therefore, the forest managers should use Eq. (4) (a type of logistic model) and Eq. (7) (a type of von Bertalanffy model). RMSE values for Eq. (4) and Eq. (7) were 2.3679 m and 2.3589 m, respectively. Lumbres *et al.* (2013) reported that the Logistic-type model was the best with an RMSE of 2.86510 m in predicting the heights of Khasi pine (*Pinus kesiya* Royle ex Gordon) species as compared with Chapman-Richards, Weibull, Schnute, Exponential and Korf/Lundqvist functions. Similar findings were reported by Abedi and Abedi (2020). In their study, the Logistic-type model gave the least prediction error with an RMSE ranged from 2.070 m to 2.162 m for the height predictions of Caucasian oak, Field maple and Hornbeam species. In contrast to these studies, in the study of Chai *et al.* (2018) the Chapman-Richards function was reported as the best model with an RMSE of 2.889 m in predicting the heights of Chinese cedar (*Cryptomeria fortunei* Hooibrenk) trees in a plantation field. Seki and Sakici (2022) recommended the use of the Chapman-Richards function having an RMSE of 1.41 m for predicting the heights of Crimean pine species. Conversely, Carus and Akguş (2018) compared a total of 20 nonlinear H-D models including the Logistic-and Richard-type models and they reported that the Chapman-Richard model gave the poorest estimations with an RMSE of 1.99 m in predicting the heights of Stone pine (*Pinus pinea* L.) in a planted forest.

Height-diameter relationships can considerably change depending on the stand conditions. For that reason, the additional covariates such as basal area and site index are required for generalizing a H-D model (Adame *et al.*, 2008; Bronisz and Mehtätalo, 2020; Sharma, 2015; Sharma and Parton, 2007; Temesgen and von Gadow, K., 2004). In our study, data was collected by randomly selecting trees without sampling stand characteristics, hence the models were developed as a function of only diameter at breast height. Therefore, the estimations to be obtained from the Eq. (4) and Eq. (7) for an Oak stand outside the study area should be evaluated cautiously.

5 Conclusion

The amount of explained variations by the nonlinear H-D models ranged from 0.7605 to 0.8065. While the goodness-of-fit statistics showed that all models developed were well fitted, some of them gave an over-prediction for trees less than 5 m and taller than 20 m in height. Logistic-and modified von Bertalanffy functions resulted in better prediction accuracy. Also, these models provided the desirable biological properties such as S-shaped curve and asymptote. The results proved that the functions can be implemented in performing satisfactory accuracy for Hungarian oak stands located in Dumanlı Forest Planning Unit, Çanakkale/Turkey.

Author contributions

FB designed and performed the statistical analysis and wrote this article. AG and OÜ contributed to review and edit it. All authors read and approved its final form.

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