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Regression-based estimates for sectional stem volume of Scots pine (*Pinus sylvestris* L.) in Greece considering the mean diameter reduction factor

Regressionsbasierte Schätzungen des abschnittsweisen Stammvolumens der Waldkiefer (*Pinus sylvestris* L.) in Griechenland unter Berücksichtigung des mittleren Durchmesserreduktionsfaktors

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Keywords: Biometry, growing stock, segmental volumetry, allometry, Pieria Mountains, Smalian formula

Schlüsselbegriffe: Biometrie, Vorrat, Teilvolumenmessung, Allometrie, Pieria-Gebirge, Smalian-Formel

Abstract

This research presents the development of a reliable system for predicting the total stem volume of Scots pine (*Pinus sylvestris* L.) trees by building models for estimating the mean diameter reduction factor (*α*). Using easily measured in-field variables as independent variables, the construction of regression models that best fit a representative sample of trees from the mountainous complex of the Pieria Mountains was investigated. The evaluation of these models was based on a set of fit criteria, including R^2_{adj}, *SEE*, *Bias*, *AIC*, *BIC*, *VIF*, and residual distribution. The results showed that the proposed model estimates (*α*) with acceptable accuracy ($SEE = 0.187$, $R^2_{adj} = 0.839$). Ultimately, the proposed system for predicting total stem volume will support more effective and sustainable forest management. By providing accurate estimates of standing wood volume based solely on stump diameter, it offers a practical, efficient tool for reliable volumetric assessment.

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Zusammenfassung

Diese Forschungsarbeit präsentiert die Entwicklung eines zuverlässigen Systems zur Schätzung des Gesamtstammvolumens von Waldkiefer (*Pinus sylvestris* L.) durch die Erstellung von Modellen zur Schätzung des mittleren Durchmesserreduktionsfaktors (*α*). Unter Verwendung von leicht im Feld messbaren Variablen als unabhängige Variablen wurde die Erstellung von Regressionsmodellen untersucht, die am besten zu einer repräsentativen Stichprobe von Bäumen aus dem Gebirgskomplex der Pieria-Berge passen. Die Bewertung dieser Modelle basierte auf einer Reihe von Anpassungskriterien, darunter R^2_{adj}, *SEE*, *Bias*, *AIC*, *BIC*, *VIF* und Residuenverteilung. Die Ergebnisse zeigten, dass das vorgeschlagene Modell (*α*) mit akzeptabler Genauigkeit schätzt ($SEE = 0.187$, $R^2_{adj} = 0.839$). Letztendlich wird das vorgeschlagene System zur Vorhersage des Gesamtstammvolumens eine effektivere und nachhaltigere Waldbewirtschaftung unterstützen. Durch die Bereitstellung genauer Schätzungen des Holzvolumens eines Bestandes, die ausschließlich auf dem Baumstumpfdurchmesser basiert, bietet es ein praktisches, effizientes Werkzeug für eine zuverlässige Volumenbewertung.

1 Introduction

Tree taper is a critical characteristic of trees that is considered when estimating their volume, biomass, or the carbon they can store (Polglase *et al.*, 2003; Jones *et al.*, 2022). Tree taper refers to the rate at which stem diameters decrease from the ground to the tree top (Burkhart & Tomé, 2012). The external profile of a tree typically shows a clear bend near the base. Then, it becomes approximately linear through the central portion of the bole, and it exhibits greater variability in the upper stem (Shaw *et al.*, 2003). Because tree boles have irregular geometric shapes, accurately estimating and predicting their volume and biomass has been a persistent challenge.

The stem of a tree can be conceptualized as the surface generated by rotating a curve that is convex outward near the lower stem and concave inward toward the upper stem. Its overall form generally lies between that of a cone and a paraboloid. Given the substantial economic implications of accurate estimates of commercial wood products, particularly stem volume, research in forest biometrics has increasingly focused on factors that improve the precision of such estimates. Usually, the largest share of tree volume and biomass is in the stem (Poorter *et al.*, 2012), and therefore, changes in taper lead to changes in the total volume and biomass of trees that are otherwise of the same size (*e.g.*, same height and diameter). For this reason, the investigation of conicity is a significant interest for most researchers involved in model development (Eckmüllner *et al.*, 2007; Weiskittel *et al.*, 2011).

Several factors have been studied and proposed to describe the shape of the stem, such as the form factor, form height, form ratio, and diameter reduction factor, along with various taper equations (Prodan, 1965; Matis, 2004; Van Laar & Akça, 2007). Nevertheless, the ability to mathematically describe the shape of tree stems has been the subject of ongoing research for more than a hundred years (Kublin *et al*., 2013; McTague & Weiskittel, 2021).

One form factor, which has not yet been extensively investigated and serves as an indicator of stem form, is the mean diameter reduction factor (α) of the tree's stem. This factor quantifies the average decrease in stem diameter per unit increase in height. It is calculated as the difference between two specified diameters, typically the stump diameter ($d_{0.3}$), measured 0.3 m above the ground, or the breast-height diameter ($d_{1.3}$), measured 1.3 m above the ground, and the diameter measured at a selected upper point on the stem, divided by the vertical distance between the measurement points (Matis, 2004). It is not constant along the stem, but it varies systematically. It is relatively high near the base, decreases toward the mid-height of the stem, and subsequently increases again as it approaches the upper portions of the stem (Matis, 2004).

Knowledge of (α), derived from appropriate taper models, is crucial for precise estimation of stem volume, which depends on the availability of diameter measurements at multiple heights. In addition, understanding taper contributes indirectly to characterizing stand structure (West, 2015) and provides essential information for effective forest management. To the best of the authors' knowledge, the existing body of research on this topic remains limited. Indicatively, Pérez *et al*. (2003) introduce a diameter reduction factor (cm of diameter per m of height) derived from stem analysis of teak (*Tectona grandis*) trees, Maraseni *et al*. (2007) estimated the taper rates of basal and distal parts through mean diameter reduction factor as well as volume of smaller-sized logs in Spotted gum (*Corymbia citriodora* subsp. *variegata*) trees, Fuentes *et al*. (2010) used a stem-diameter reduction factor for stem diameters calculation in European beech (*Fagus sylvatica* L.) trees, while Diamantopoulou (2022) and lately Diamantopoulou (2025) compared non-linear regression and advanced machine learning modeling approaches for fir (*Abies borisii-regis* Matff.) trees mean diameter reduction factor reliable prediction.

For many decades, despite its limitations, regression modeling (Draper & Smith, 1998) has been widely used in forest research, providing solutions for both research and practice. For this reason, developing a volumetric modeling system that is easy to understand and apply by forest practitioners remains essential.

The aim of this study is to develop a robust regression-based model to predict the mean diameter reduction factor (α) using field measurements from Scots pine (*Pinus sylvestris* L.) stands in the Pieria Mountains. A further objective is to assess whether the model's predictions of each tree's mean diameter reduction factor can be reliably applied to estimate total stem volume accurately. Finally, we aim to provide a precise and trustworthy system for stem-volume prediction of Scots pine trees, easily understandable and usable, based solely on $d_{0.3}$, an easily obtainable field stem diameter that remains accessible even after a tree has been felled.

2 Materials and Methods

2.1 Study area and sample data

The study area consists of the Public Forest Complex of Skoteina-Livadi-Fteri-Platanorema in the Pieria Mountains, with a total area of 9957.68 ha, of which 1272.08 ha are in the pine (Black pine and Scots pine) management class, of which980.73ha are forested (Pieria Forest Directorate, 2013) (Figure1). The local population of *Pinus sylvestris* L. constitutes the southernmost forest boundary of its geographical distribution in Eastern Europe (Zagas, 1990; Gerasimidis *et al*., 2006; Grigoriadis *et al*., 2014; Oikonomakis & Ganatsas, 2020).

The specific description of this forest complex enters into a more detailed ecological description of the individual forest stands. As regards the specific description of the stands from which the sample was taken, there is some information through the description sheets regarding the existence of silvicultural species, silvicultural shape, tree's age and regeneration dynamics (Pieria Forest Directorate, 2013), which indicatively attempt to analyze the assessment of the current situation in terms of silvicultural-management measures and proposed adaptation strategies to the current conditions (Peterson St-Laurent *et al*., 2021), as well as the prediction of their future evolution over time in the context of climate change (Mauri *et al*., 2022). Mainly, mixed forest stands of Scots pine (*Pinus sylvestris* L.) with Black pine (*Pinus nigra* Arn.) and sporadic European beech (*Fagus sylvatica* L.) trees are observed, a fairly common occurrence for mixed forests in Greece (Zagas *et al*., 2001), creating uneven-aged stands with a sub-forested silvicultural shape and a range of pine ages of approximately 1-100 (120) years. The regeneration dynamics of pine are low to moderate by positions, as a gradual ingress of beech is observed (Pieria Forest Directorate, 2013).

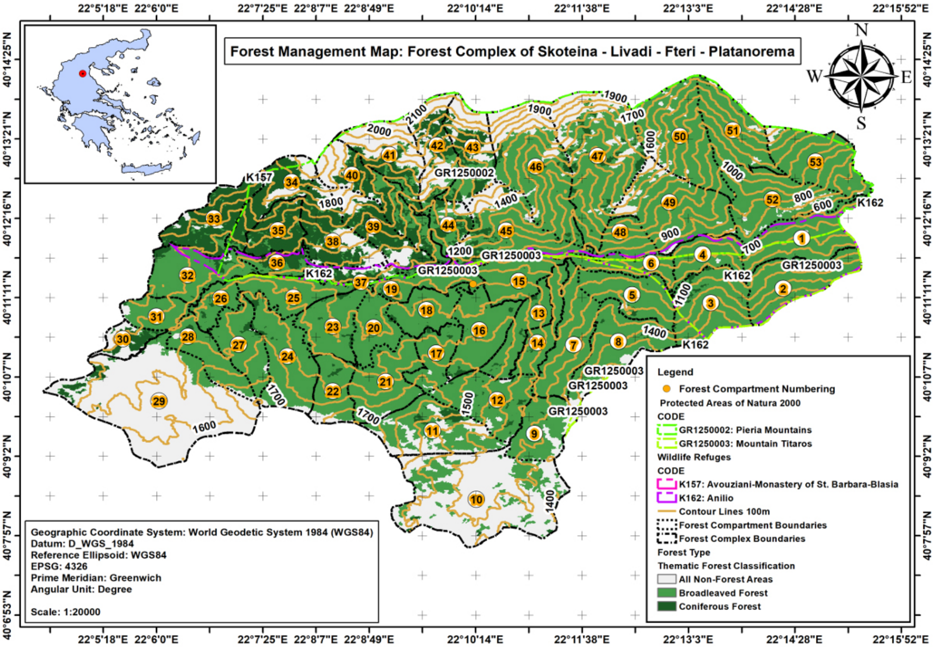

Figure 1: The study area of the Public Forest Complex of Skoteina - Livadi - Fteri - Platanorema in the Pieria Mountains.

Abbildung 1: Das Untersuchungsgebiet im Gebiet Skoteina–Livadi–Fteri–Platanorema in den Pieria-Bergen.

In the context of this study, a simple random pilot sample of 10 trees was collected from the study area, through which a pilot regression equation was developed. The standard error of estimate of the pilot equation (Brooks & Barcikowski, 1994) led to the simple random sample of 100 Scots pine (*Pinus sylvestris* L.) trees to be measured in the field, which fully satisfied the requirements for the accuracy of the regression equations developed.

For the trees in the sample, the $d_{0.3}$ and the $d_{1.3}$ were measured to the nearest 0.1 cm using a Finnish caliper. Total tree height (*tht*) was recorded to the nearest 0.1 m with a Blume–Leiss hypsometer. Stem diameters were then measured at 2-m intervals along the stem, beginning at $d_{1.3}$, using a Spiegel relascope. All primary data were collected between early and late May.

The most commonly used general formula, and also applied in this study, refers to the total stem. Accordingly, the mean diameter reduction factor (a) for each tree was calculated using the formula (Prodan, 1965; Matis, 2004):

$$a = \frac{d_{0.3} - d_n}{L} \qquad (1)$$

where: $d_{0.3}$ is the stump diameter, d_n is the last upper diameter of the stem, L is the distance between the diameters $d_{0.3}$ and d_n.

The specific limits on the diameters in the formula depend on the research context. The mean diameter reduction factor (a) for the entire tree stem was calculated to estimate the total tree stem volume. In this context, the largest diameter considered was the stump diameter, which remains in the forest after logging, while the upper diameter approached zero. Thus, this study addresses the mean diameter reduction factor (a) for the whole tree stem.

The sample data were explored through Exploratory Data Analysis (*EDA*), in IBM SPSS Statistics v. 29.0 statistical package (IBM SPSS, 2023; Diamantopoulou, 2024). *EDA* was used to assess the structure and composition of the data set under study, using graphical representations and statistical tests (Tukey, 1977; Hartwing & Dearing 1979; Hoaglin *et al*., 1985).

The graphical representations provided a rough indication of whether the population followed a normal distribution. Several more objective formal statistical methods and tests are available to evaluate the same question. One of the most commonly used is the Lilliefors test (Lilliefors, 1967), which is a modified version of the Kolmogorov–Smirnov test designed for situations in which the mean and variance of the normal distribution population are unknown. This modification was used as a non-parametric goodness-of-fit test to assess how well the a, which served as the dependent variable in the regression models, matches the normal distribution (H_0) while exploratory graphing was used to assess homogeneity or non-homogeneity of variance for a.

Furthermore, collinearity among the independent variables ($d_{0.3}$, h) was examined using the variance inflation factor (*VIF*):

$$VIF = \frac{1}{1 - R_j^2} \qquad (2)$$

where: R_j^2 is the coefficient of determination from regressing Xj on all the other predictors.

2.2 Model development

To develop models of the a, standard regression methods (simple, multiple, and nonlinear) were used, applying the available procedures in IBM SPSS Statistics v. 29.0 (IBM SPSS, 2023).

The Levenberg-Marquardt iterative method (Levenberg, 1944; Marquardt, 1963; Ratkowsky, 1990; Murthy, 2014) was used to fit the non-linear equations to the data. The selection of this methodology is based on combining the best features of two methods: the Gauss-Newton method and the steepest descent method, avoiding their most serious limitations. It is quite likely to converge almost always (IBM SPSS, 2023), while it is rarely found to recalculate the model coefficients indefinitely, as is often the case with the steepest descent method. Overall, this method seems to work well in many instances of forest biometric variable estimation (Diamantopoulou, 2022) making it a reasonable choice in practice (Draper & Smith, 1998). The sum of squared errors was utilized as the loss function. Finally, initial parameter values for the non-linear model estimation were determined following the procedures outlined in Draper and Smith (1998).

2.3 Measures for evaluating regression models

The evaluation of the regression models were based on the following adjustment criteria (Raptis *et al*., 2024):

a) the adjusted coefficient of determination (R^2_{adj})

$$R^2_{adj.} = 1 - \frac{(1-R^2)(n-1)}{n-p-1} \qquad (3)$$

b) the standard error of estimate (*SEE*)

$$SEE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y_i})^2}{n-p-1}} \qquad (4)$$

c) the *Bias*

$$Bias = \frac{\sum_{i-1}^{n}(y_i - \widehat{y_i})}{n} \qquad (5)$$

d) Akaike information criterion (*AIC*) for least-squares regression with normally distributed errors

$$AIC = n\,ln\left(\frac{SSE}{n}\right) + 2(p+1) \qquad (6)$$

e) Bayesian information criterion (*BIC*) for least-squares regression

$$BIC = n\,ln\left(\frac{SSE}{n}\right) + (p+1)ln\,(n) \qquad (7)$$

where: R^2 is the coefficient of determination, y_i and $\hat{y}_i$ are the values of the observed and predicted values of the dependent variable, respectively, n is the number of observations, p is the number of independent variables in the equation.

2.4 Volume estimation

Segmented volume was calculated using Smalian's formula (Equation 8). This formula is a widely recognized one in forest biometrics for estimating the volume of logs or tree stems with irregular shapes (Philip, 1994; Matis, 2004; Van Laar & Akça, 2007). This method ensures accurate stem volume calculation, accounting for the conical shape of tree stems, which is significant for irregularly shaped trees (Diamantopoulou & Georgakis, 2024):

$$V = \frac{\pi}{4} \cdot l \cdot \left(\frac{{d_{0.3}}^2}{2} + {d_{1.3}}^2 + {d_2}^2 + {d_3}^2 + \cdots + {d_{n-1}}^2 + \frac{{d_n}^2}{2} \right) + \frac{1}{3} \cdot \frac{\pi}{4} \cdot {d_n}^2 \cdot l_k \quad (8)$$

where: $d_{0.3}$ is the stump diameter measured 0.3 m above the ground, $d_{1.3}$ is the breast height diameter measured at 1.3 m from the ground, d_n is the last upper diameter of the stem, $d_{1,2,3,\ldots n}$, are the sequential stem diameters with 2 m step, l_k is the length of the remaining segment between the last observed diameter and the tree top.

3 Results

Based on the exploratory data analysis (Figure 2), the box plot of the dependent variable *α* (Figure 2a) shows no isolated or extreme outliers. The median is centered within the interquartile range, suggesting an approximately symmetrical distribution. The data spread is relatively large, ranging from about 0.3 cm/m to 2.5 cm/m. The histogram of the observations (Figure 2b) satisfactorily approximates the normal distribution, as indicated by the black continuous line. The normal probability plot (Figure 2c) shows the adjustment of the mean diameter reduction factor values to the normal distribution, as the observations are shown to coincide or tend towards the straight line of the graph (Figure 2c), with some minor losses at the ends. Finally, the normal deviation plot (Figure 2d) complements the Q–Q plot by explicitly illustrating the deviations of the observed values from those expected under normality.

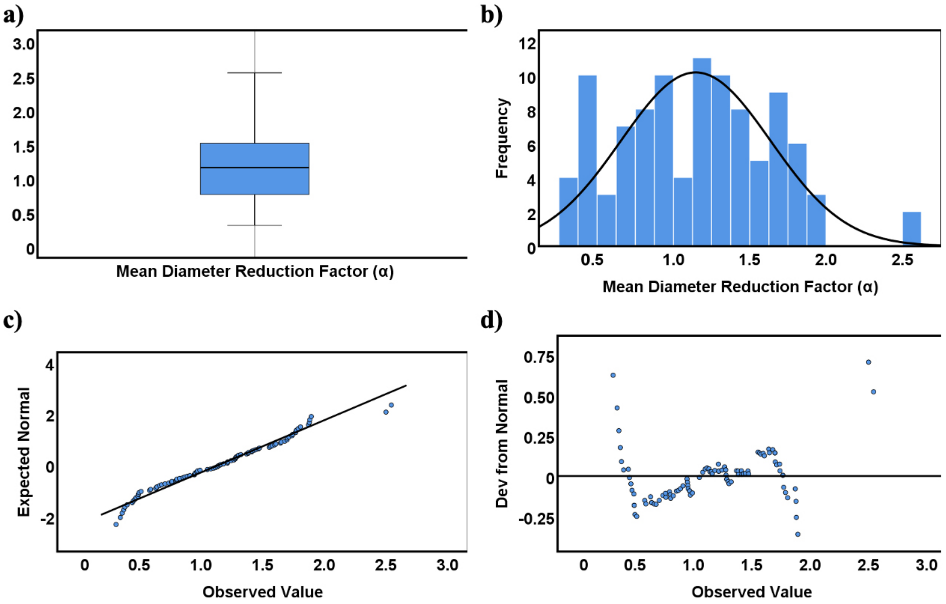

Figure 2: a) Box plot, b) Histogram, c) Normal probability plot and d) Deviation from normal (Q-Q) plot, as resulted from the EDA of the mean diameter reduction factor(α).

Abbildung 2: a) Boxplot, b) Histogramm, c) Normalverteilungsdiagramm und d) Abweichung vom Normalverteilungsdiagramm (Q-Q-Diagramm), wie sie sich aus der EDA des mittleren Durchmesserreduktionsfaktors (α) ergeben.

In order to have an objective evaluation of whether the α follows a normal distribution at the 5% significance level, the modified Kolmogorov-Smirnov test of normality was applied. According to the results presented in Table 1, the significance value (Sig.) is 0.200, which exceeds the 0.05 threshold. Therefore, there is insufficient evidence to reject the null hypothesis of normality at $\alpha = 0.05$, suggesting that the variable is normally distributed (Table 1).

Table 1: Kolmogorov-Smirnov normality test of the dependent variable of the mean diameter reduction factor (α).

Tabelle 1: Kolmogorov-Smirnov-Test zur Überprüfung der Normalverteilung der abhängigen Variable des mittleren Durchmesserreduktionsfaktors (α).

Dependent Variable: Mean Diameter Reduction Factor(α)		
Test of Normality: Kolmogorov-Smirnov		
Statistic	df	Sig.
0.052	100	0.200^{*}

The size of diameters at sequential heights, the total height and the mean diameter reduction factor, the range of the variable values, the minimum and the maximum values the arithmetical mean and the standard error of the mean, the standard deviation and the variance of the sample variables are given in Table 2.

Table 2: Descriptive statistics of the variables.

Tabelle 2: Deskriptive Statistiken der Variablen.

Variable	N	Range	Minimum	Maximum	Mean		Std. Deviation	Variance
					Statistic	Std. Error		
$d_{0.3}$,cm.	100	83.6	12.7	96.3	49.923	1.867	18.673	348.683
$d_{1.3}$, cm.	100	68.7	8.7	77.4	32.953	1.597	15.973	255.142
$d_{2.3}$, cm.	100	69.0	5.3	74.3	31.317	1.540	15.408	237.428
$d_{4.3}$, cm.	100	65.8	1.3	67.1	28.666	1.489	14.897	221.940
$d_{6.3}$, cm.	98	60.0	2.9	62.9	26.683	1.409	13.950	194.617
$d_{8.3}$, cm.	97	55.3	2.3	57.6	24.261	1.342	13.219	174.767
$d_{10.3}$, cm.	96	51.5	1.9	53.4	21.836	1.303	12.774	163.181
$d_{12.3}$, cm.	90	47.1	1.4	48.5	20.090	1.225	11.628	135.209
$d_{14.3}$, cm.	83	42.5	0.7	43.2	18.480	1.157	10.543	111.167
$d_{16.3}$, cm.	77	37.0	1.2	38.2	16.447	1.085	9.523	90.690
$d_{18.3}$, cm.	71	32.0	1.7	33.7	14.198	1.009	8.503	72.310
$d_{20,3}$, cm.	61	28.3	1.6	29.9	12.164	0.943	7.370	54.324
$d_{22.3}$, cm.	47	24.5	1.1	25.6	11.045	0.900	6.175	38.141
$d_{24.3}$, cm.	34	18.5	2.3	20.8	10.076	0.731	4.265	18.194
$d_{26.3}$, cm.	25	15.1	1.6	16.7	7.216	0.696	3.482	12.126
$d_{28.3}$, cm.	13	10.0	1.3	11.3	6.154	0.732	2.642	6.981
$d_{30.3}$, cm.	5	5.3	3.9	9.2	5.720	0.941	2.105	4.432
$d_{32.3}$, cm.	1	0	6.9	6.9	6.900	-	-	-
h, m.	100	28.1	4.7	32.8	21.101	0.618	6.1825	38.223
a, cm./m.	100	2.2	0.3	2.5	1.146	0.049	0.490	0.240

The relationship between the mean diameter reduction factor and the stump diameter and the total tree height (Figure 3) cannot be reliably classified as linear or non-linear. For this reason, a wide range of equations was fitted to the data using regression analysis (linear, multiple, and non-linear), to select the equation that best fit the data with the greatest possible accuracy.

The models that provided the best fit to the data for each regression category are given in Table 3.

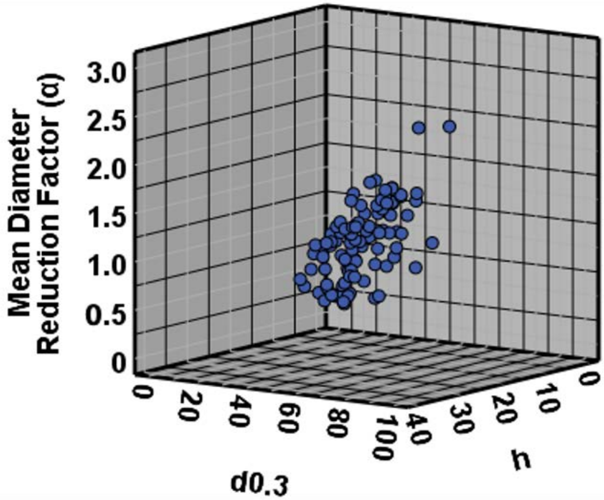

Figure 3: Scatter plot of the mean diameter reduction factor- stump diameter-total height.

Abbildung 3: Streudiagramm des mittleren Reduktionskoeffizient von Durchmesser – Baumstumpfdurchmesser – Gesamthöhe.

The three best models of simple linear (Table 3, Equation 9), multiple linear (Table 3, Equation 10), and non-linear regression (Table 3, Equation 11) gave similar results in terms of their evaluation measures.

Table 3: Fitting criteria of the regression models.

Tabelle 3: Bewertungskriterien der Regressionsmodelle.

Dependent Variable: Mean Diameter Reduction Factor (α)					
Model Summary					
α/α Equation	R^2_{adj}	*SEE*	*AIC*	*BIC*	*Bias*
(9) $b_0 + b_1 \cdot \sqrt{{d_{0.3}}^2 h}$	0.837	0.1981	-321.814	-316.603	-0.00150
(10) $b_0 + b_1 \cdot d_{0.3} + b_2 \cdot h^2$	0.839	0.1868	-319.374	-314.163	-0.08464
(11) $b_1 - b_2 \cdot \exp\left[-b_3 \cdot \left({d_{0.3}}^2 h\right)^{b4}\right]$	0.837	0.1987	-325.484	-317.668	-0.00015

The multiple linear model that was examined and ultimately selected as the best fit for the field data (Table 3, Equation10) produced an adjusted coefficient of determination $R^2_{adj} = 0.839$ and a standard error of estimation $SEE = 0.1868$. The *VIF* values were also calculated for the coefficients b_1, b_2, and b_3 of Equation 10. Their *VIF* values were less than 5, indicating no serious multicollinearity in the model (Marquardt, 1974; Neter *et al*., 1990). This condition can be acceptable in the sense that the model coefficient estimates are not significantly affected by limited multicollinearity and are therefore considered satisfactorily stable.

Table 4: Parameter estimates of the multiple linear regression model (10).

Tabelle 4: Schätzungen der Parameter des multiplen linearen Regressionsmodell (10).

Dependent Variable: Mean Diameter Reduction Factor (α)									
	Parameter Estimates			Std. Error		95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	VIF	
Equation	b_0	b_1	b_2	b_1	b_2	$b_1 b_2$	$b_1 b_2$	b_1	b_2
$b_0 + b_1 \cdot d_{0.3} + b_2 \cdot h^2$	0.013	0.016	0.001	0.002	0	0.012 0	0.019 0.001	2.644	2.644

The histogram of the residuals of the best-performing multiple linear equation (Table 3, Equation 10) satisfactorily approximates the normal distribution (Figure 4a). Furthermore, the distribution of the residuals of the estimated mean diameter reduction factor showed a stable variance across the range of predicted values (Figure 4b). Figure 4c further supported a very good fit of the model to the data, yielding estimates that explain approximately 83% of the total variance of the observed values of α.

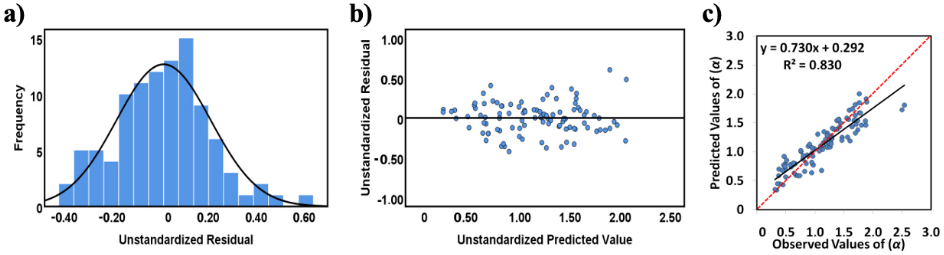

Figure 4: Measures of the multiple linear regression model (10), evaluation in the form of a) histogram, b) residual distribution graph and c) 45-degree line plot.

Abbildung 4: Bewertungsmaßnahmen des multiplen linearen Regressionsmodells (10), in Form von a) Histogramm, b) Residuenverteilungsdiagramm und c) 45°-Linien-Diagramm.

3.1 Volume prediction

To further evaluate the effect of the a estimation by the model of Table 4 on the configuration of the stem diameter estimates, these diameters were used as data for the segmental calculation of stem volume using Smalian‘s formula (8). The observed total stem volume, calculated from the observed stem diameters, was compared with the estimated volume calculated with Smalian‘s formula (8) using the stem diameters produced from the estimated a.

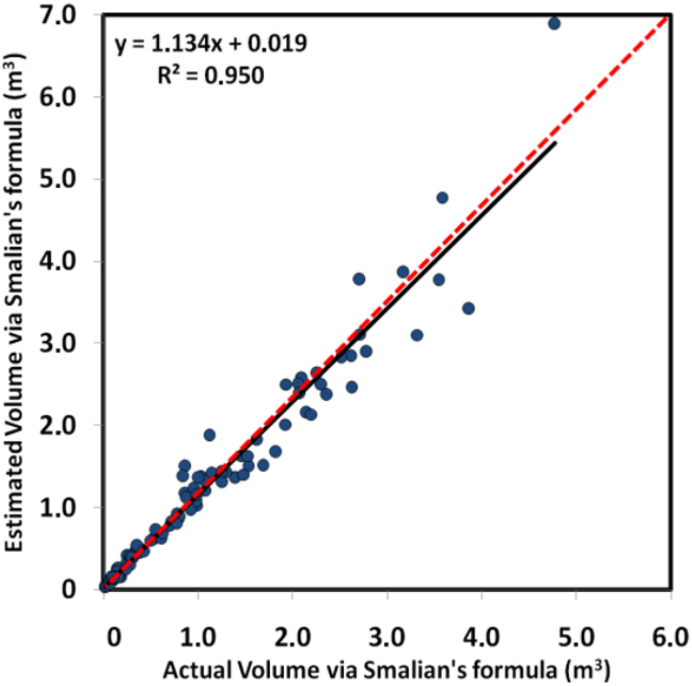

Figure 5: Scatterplot of the actual volume via the Smalian's formula with the estimated volume via the Smalian's formula (8).

Abbildung 5: Streudiagramm der 45°-Linie des tatsächlichen Volumens mit der Smalian-Formel mit des geschätzten Volumens mit der Smalian-Formel mit Durchmesserdaten, die sich aus der Anwendung der Gleichung (8) ergeben haben.

Based on the 45-degree line scatter plot (Figure 5), the very good indirect estimation of the segmental total stem volume is highlighted. The coefficient of determination for the multiple linear relationship between the actual and indirectly calculated total stem volumes was particularly high, at 0.955. The standard error of the estimate for this relationship between total stem volumes was found to be 0.271, indicating that the estimated total volume is slightly overestimated and follows the dispersion of actual volume values.

From the comparative evaluation of the estimated versus actual stem volume, by stump diameter per tree, it was concluded that the estimates of total stem volume are highly accurate in all stump diameter tree-by-tree examined (Figure 6).

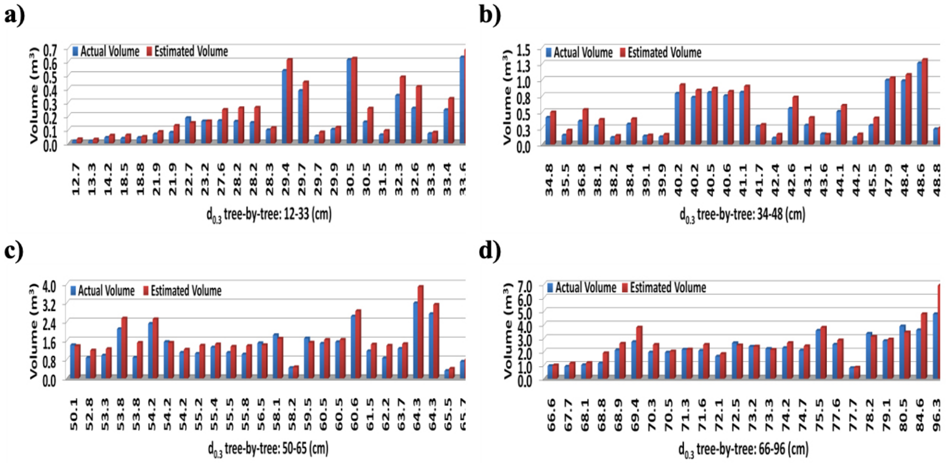

Figure 6: Comparative evaluation of actual volume using the Smalian's formula (8) in relation to the indirectly calculated volume using the multiple linear equation (10), based on the stump diameter of a) tree-by-tree: 12-33 cm, b) tree-by-tree: 34-48 cm, c) tree-by-tree: 50-65 cm, d) tree-by-tree: 66-96 cm.

Abbildung 6: Vergleichende Bewertung des tatsächlichen Volumens unter Verwendung der Smalian-Formel (8) im Verhältnis zum indirect berechneten Volumen unter Verwendung der multiplen linearen Gleichung (10), basiert auf dem Baumstumpfdurchmesser von a) Baum zu Baum: 12–33 cm, b) Baum zu Baum: 34–48 cm, c) Baum zu Baum: 50–65 cm, d) Baum zu Baum: 66–96 cm.

The statistical analysis of the actual volume values yielded a mean value of 1.078 m^3, with a standard error of the mean equal to 0.104, while the standard deviation of the values was found to be equal to 1.041 (Table 5), whereas the estimated volume values yielded a mean value of 1.242 m^3, with a standard error of the mean equal to 0.121, while the standard deviation of the values was found to be equal to 1.211 (Table 5).

Table 5: Descriptive statistics of actual and estimated volume values.

Tabelle 5: Deskriptive Statistiken der tatsächlichen und geschätzten Volumenwerte.

Variable	N	Range	Minimum	Maximum	Mean		Std. Deviation	Variance
					Statistic	Std. Error		
Actual Volume,m^3	100	4.760	0.014	4.774	1.078	0.104	1.041	1.084
Estimated Volume,m^3	100	6.857	0.027	6.884	1.242	0.121	1.211	1.468
Act.Vol.(12-33cm.),m^3	25	0.616	0.014	0.630	0.185	0.036	0.181	0.033
Est.Vol.(12-33cm.),m^3	25	0.658	0.027	0.685	0.233	0.040	0.201	0.041
Act.Vol.(34-48cm.),m^3	25	1.163	0.093	1.257	0.458	0.067	0.336	0.113
Est.Vol.(34-48cm.),m^3	25	1.177	0.133	1.310	0.534	0.070	0.350	0.123
Act.Vol.(50-65cm.),m^3	26	2.867	0.306	3.174	1.410	0.137	0.699	0.490
Est.Vol.(50-65cm.),m^3	26	3.464	0.400	3.864	1.618	0.151	0.772	0.597
Act.Vol.(66-96cm.),m^3	24	4.000	0.773	4.774	2.295	0.206	1.013	1.027
Est.Vol.(66-96cm.),m^3	24	6.073	0.811	6.884	2.623	0.265	1.302	1.697

After applying the mean diameter reduction factor, which resulted from the development of the best-performing multiple linear model (Equation 10) to estimate the total stem volume, the corresponding means (standard error of the mean) of tree-by-tree were calculated for $d_{0.3}$ of 12 to 33 cm as 0.233 (0.040), for $d_{0.3}$ of 34 to 48 cm as 0.534 (0.070), for $d_{0.3}$ of 50 to 65 cm as 1.618 (0.151) and for $d_{0.3}$ of 66 to 96 cm as 2.623 (0.265), respectively (Table 5). In absolute values, the incorporation of the multiple linear equation (Equation 10) in the estimation of the stump diameters of 66 to 96 cm appears to overestimate the mean of the actual volume. Less overestimation was found in the estimation of stump diameters of 50 to 65 cm. In comparison, even less overestimation was observed when the multiple linear model (Equation 10) was incorporated into the estimation of stump diameters of 34 to 48 cm and 12 to 33 cm.

The coefficient of determination (standard error of the estimate) of the estimated volume values for $d_{0.3}$ of 12 to 33 cm with the incorporation of the multiple linear model (Equation 10) was found to be equal to 94.9% (0.046), the corresponding coefficient of determination of the estimated values for $d_{0.3}$ of 34 to 48 cm was found to be equal to 98.3% (0.046), the estimated volume values for $d_{0.3}$ of 50 to 65 cm it was found to be equal to 91.9% (0.223) and finally, the estimated volume values for $d_{0.3}$ of 66 to 96 cm, it was found to be equal to 85.2% (0.511), which shows that the estimation of the of the stump diameters values followed the dispersion of the actual volume values (Figure 7) after incorporating of the multiple linear equation (Equation 10).

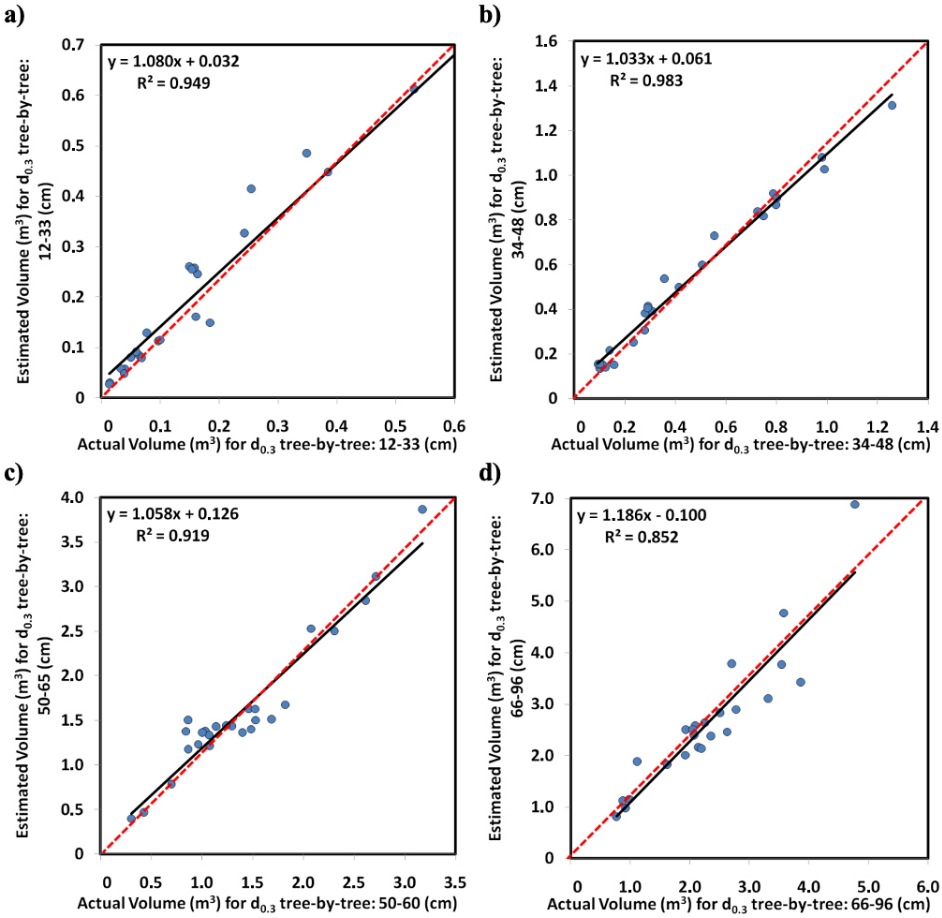

Figure 7: Scatter plot of the actual volume via the Smalian's formula (8) with the estimated volume via the Smalian's formula (8) for each stump diameter of a) tree-by-tree: 12-33 cm, b) tree-by-tree: 34-48 cm, c) tree-by-tree: 50-65 cm, d) tree-by-tree: 66-96 cm.

Abbildung 7: Streudiagramm des tatsächlichen Volumens nach der Smalian-Formel (8) mit dem geschätzten Volumen nach der Smalian-Formel (8) für jeden Baumstumpfdurchmesser von a) Baum zu Baum: 12–33 cm, b) Baum zu Baum: 34–48 cm, c) Baum zu Baum: 50-65 cm, d) Baum zu Baum: 66-96 cm.

4 Discussion

In recent years, advances in the combined use of non-linear mixed effects models and segmented polynomial taper functions (Saygili & Kahriman, 2023), as well as the integration of advanced machine learning techniques such as generalized regression, Bayesian regularization, and support vector regression (Diamantopoulou, 2025), have significantly improved the accuracy of volume, diameter, and taper profile estimates. Overall, these approaches allow the development of more accurate and adaptive models with the ability to reliably estimate volume at different heights along the stem segments and by diameter classes, ensuring a more comprehensive understanding of the stem structure and the productive potential of the forest, providing essential tools for forestry applications such as logging planning, growth modeling, and timber utilization. However, forestry practice needs to acquire skills, expertise, and qualifications that it has not yet developed in order to apply and utilize these advanced modeling methodologies. This study aims to provide a system for stem-volume prediction of Scots pine (*Pinus sylvestris* L.) trees, with acceptable precision, easily understandable and usable, based on well-known regression modeling.

4.1 Model development

Tree stump diameter and total tree height were the independent variables used to develop the stem mean diameter reduction factor model. As Figure 3 revealed, the relationship between the mean diameter reduction factor and the stump diameter and the total tree height could not be reliably classified as linear or non-linear. Therefore, numerous equations were adjusted to the data using regression analysis, including simple, multiple, and non-linear models, to select the most appropriate model. Finally, the multiple linear model (Equation 10) estimated the mean diameter reduction factor more accurately, as it was well adjusted to the data, with an adjusted coefficient of determination of 0.839 and a standard error of estimate equal to 0.1868 (Table 3).

Accurate estimation of the mean diameter reduction factor enables the accurate prediction of diameters along the tree stem. The most valuable product in Greece, based on the selling price, is telecommunications and electricity poles, which, when they appear in the shaped stem and in any diameter (from 10 to 50 cm), maximize the commercial value of the Scots pine (*Pinus sylvestris* L.) tree (Kontos, 2002). This is a critical factor, as accuracy in this range of diameters directly affects estimates of productivity, economic performance, and forest management planning (Avery & Burkhart, 2015; Ulak *et al.*, 2022), *i.e.*, in the lower sections where the largest percentage of marketable wood volume is concentrated (Saygili & Kahriman 2023).

4.2 Volume estimation

The results of the mean diameter reduction factor of Equation (10) were used in Smalian's segmental volumetry (Equation 8) for the indirect calculation of the total stem volume of Scots pine (*Pinus sylvestris* L.) in the Pieria Mountains of Greece, using a sample size that exceeded the minimum sampling requirements by approximately 1.5 times.

The results of the estimation of stem volume at all diameters of the stem per tree (Figure 6) indicate relatively high accuracy (Figure 7) with the incorporation of the applied multiple linear model (Equation 10), which estimates a quite well, allowing such equations to be reliably used in forest management (McTague & Weiskittel, 2021; Hansen *et al.*, 2023). A comparison of the standard errors of the mean of their estimated volumes per tree (Table 5) shows that the values are slightly overestimated for stump diameters from 12 to 33 cm with $SEE = 0.040$ and from 34 to 48 cm with $SEE = 0.070$, compared to the corresponding standard errors of the means of their actual volumes at the same stump diameters.

Based on the 45-degree line scatter plots (Figure 7), which were compiled for the overall comparative evaluation of the actual versus estimated total volume overall and tree-by-tree, as well as the calculation of the standard error of their estimates, it is found that the stem volume estimates obtained by incorporating the information from the multiple linear equation (Equation 10) have an $SEE = 0.271$ m³, slightly overestimating the actual volume values from the smallest to the largest stump diameters tree-by-tree. To predict the stem volume for stump diameters from 12 to 33 cm and from 34 to 48 cm per tree, were calculated $SEE = 0.046$ m³, followed by a significant and smaller overestimation of values across the entire range of stump diameters in the sample, thus contributing to the most reliable and accurate indirect determination of the total volume per tree of all these stump diameters. Also, a slightly greater overestimation was found in stump diameters from 50 to 65 cm and from 66 to 96 cm per tree, presenting $SEE = 0.223$ m³ and $SEE = 0.511$ m³, respectively, which values are considered equally acceptable. This distinction between stem dimensions highlights differences in morphology, conicity, and standing wood volume (Van Laar & Akça, 2007). The qualitative distribution of volume per tree provides a more targeted description of the structure of the forest and a more accurate representation of production, which is particularly useful for planning logging operations and economic evaluation of the forest (Pretzsch, 2009).

The importance of low standard errors of the volume estimates in the lower stem cannot be misrecognized. Given that most of the merchantable wood volume and economic value is concentrated in the first part of the stem, high accuracy in this area makes the multiple linear model (Equation 10) reliable and applicable to forest management decisions (Husch *et al.*, 2003). In contrast, higher standard errors of the estimates in the upper parts of the stem have a limited effect, as the wood in these parts has a smaller diameter, lower quality, and limited commercial value. Based on this, high accuracy in the lower stem is considered desirable and contributes to increased confidence in total volume estimates (Kozak, 2004). The first section of the stem (the lower stem) accounts for most of the marketable wood volume and economic value, as confirmed by Saygili and Kahriman (2023). Thus, reducing the standard error of the mean (Table 5) in this section is crucial, as lower deviations lead to more accurate income forecasts, optimized logging operations, and reduced economic losses in timber marketing (Avery & Burkhart, 2015). In contrast, the higher standard error of the mean (Table 5) in the upper parts of the stem has a reduced impact, as these usually yield smaller diameters and lower-value products (McTague and Weiskittel, 2021).

4.3 Need for reliable stem-volume prediction system

Standing wood volume is one of the most basic characteristics of forest stands, as it is the main parameter described in forest inventories and management studies. The volume of a tree depends on its height, its basal area, its shape, and the thickness of its bark (Van Laar & Akça, 2007). Therefore, estimating volume is a difficult process in-field conditions, and any error in estimating one of the above factors results in an error in the estimated volume of the stem. Consequently, field work involves calculating the three basic parameters of the geometric formula (FAO, 1981), which may lead to errors.

To avoid such errors in forestry practice, the use of volume tables is preferred. The creation of volume tables is based on the assumption that a tree‘s volume can be estimated using equations that relate it to specific tree characteristics. The breast height diameter, height, and some characteristics of the stem's shape are usually the independent variables used to determine the volume (dependent variable) of the tree. However, research shows that adding a third independent variable, such as breast height diameter and total stem height, *e.g.*, the crown base height (*cbh*) (Näslund, 1947) or the stump diameter ($d_{0.3}$) (Wagner, 1982) or the diameter at 7 m, reduces the percentage of unexplained variance and enables a more accurate estimate of tree volume. According to Van Laar and Akça (2007), research regarding the form of trees in Finland showed that a double-entry volume table for Scots pine (*Pinus sylvestris* L.) produced biased volume estimates. The addition of an extra diameter to the upper part of the tree was necessary to obtain unbiased volume estimates for these areas. Van Laar and Akca (2007) further report that forest inventory research in Germany showed that adding an additional diameter of the upper part of the stem does not significantly improve the accuracy of tree estimates.

Volume tables for Scots pine (*Pinus sylvestris* L.) of the Public Forest Complex of Skoteina-Livadi-Fteri-Platanorema in the study area of the Pieria Mountains of Greece, and for similar neighboring forests, do not exist, neither local nor general. Therefore, the need for estimates for sectional stem volume of Scots pine (*Pinus sylvestris* L.) is of vital importance.

Pieria Forest Directorate (2013) estimated standing wood volume by applying systematic sampling with sample plots across the forested area of all stands. Then, for the trees in the sample plots, diameter was measured to the nearest 0.1 cm with a caliper, and total tree height was recorded to the nearest 0.1 m with a relascope. The form factor, which was obtained according to the form factor tables provided by the

Ministry of Agriculture for each diameter class. Finally, the volume of each stem is calculated based on the geometric formula.

The suggested stem-volume prediction system that can reliably predict the total and sectional tree-by-tree volume based on the mean diameter reduction factor predictions (Equation 10) for deriving stem diameters along the stem which finally used for the segmented and total stem volume calculation using the Smalian's formula, allows for more detailed and targeted silvicultural treatment of the stem.

To further compare the effect produced from the *α* estimation (Equation 10) by the multiple linear model of Table 4 on the configuration of the stem diameter estimates, were used as data for the Smalian's segmental volumetry, the estimated volume calculated with Smalian's formula (Equation 8), was compared with the observed total stem volume, calculated with geometric formula from the observed stem diameters.

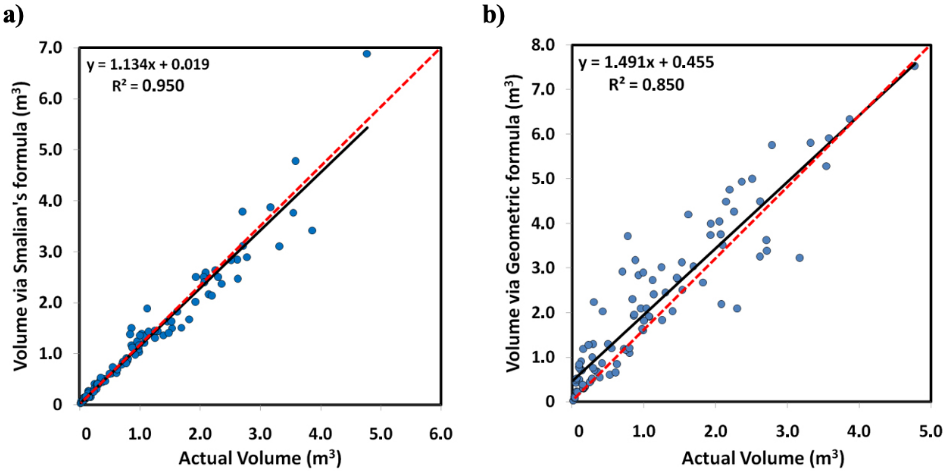

Figure 8: Scatter plot of the actual volume versus the volume via a) the Smalian's formula (8) and b) the geometric formula.

Abbildung 8: Streudiagramm des beobachteten Volumens im Vergleich zum Volumen gemäß a) der Smalian-Formel (8) und b) der geometrischen Formel.

Based on the 45-degree line scatter plots (Figure 8), the very good indirect estimation of the total stem volume is achieved with the calculated volume using Smalian's Formula (Figure 8a), compared with the geometric formula (Figure 8b), which is less accurate. The coefficient of determination for the relationship between actual volume and stem volume using Smalian's formula (Equation 8) with diameter values derived by equation (10) embedded, was particularly high at 0.950, as the standard error of the estimate for this relationship between total stem volumes was found to be 0.271, indicating that the estimated volume is slightly overestimated. On the other hand, the coefficient of determination for the relationship between actual volume and stem volume using the geometric formula was lower at 0.850, while the standard error of estimation for the relationship between total stem volumes was 0.653, indicating that the estimated volume is significantly overestimated. This finding is of high practical significance since the management of the study area is currently based on the geometric formula.

Stump-based models are essential for reconstructing tree dimensions when d1.3 measurements are unavailable due to illegal logging or natural disturbances (Di Cosmo & Gasparini, 2020). A model to estimate $d_{1.3}$ of felled trees was proposed by the first Italian National Forest Inventory in 1985 (MAF-ISAFA 1988), which classified the two broad groups of forest types and used $d_{0.3}$ as the only quantitative variable. Subsequent analyses using data from multiple species demonstrated that incorporating both species specification and $d_{0.3}$ substantially improves $d_{1.3}$ prediction, with accuracy gains exceeding 30% (Di Cosmo & Gasparini, 2020). The competition of individual trees for natural resources leads in growth of their height as well as their basal area (Canham & Uriarte, 2006), while height growth is mainly determined by environmental conditions and growth of basal area is effectively constrained by growing space (Tomé & Burkhart, 1989). According Garlemos *et al.* (2019) in three study areas in Northern Greece using generalized linear modeling that integrates stump height diameter distance to neighboring trees, and tree species significantly enhances basal area predictions in mixed pine and spruce forests. However, incorporating neighbor effects reduced prediction precision in pure beech forests. This contrast suggests that stand composition determines the effectiveness of competition-based models.

The approach for predicting the mean diameter reduction factor can offer a solution of high practical importance. Once the (α) is determined, then the stump diameter measurement is the only diameter measurement required for assessing both the segmental and the total stem volume. This approach reduces the error of the geometric formula currently used in the forest by 0.382 m^3 (58.4%), thereby contributing to the most reliable and accurate indirect determination of total stem volume tree-by-tree. The proposed strategy can provide the most precise information about the total stem volume, utilizing the easiest to obtain stem diameter, which is the stump diameter.

5 Conclusions

The proposed model for estimating the mean diameter reduction factor provides a simple, easy-to-use solution for accurately predicting the total stem volume of Scots pine (*Pinus sylvestris* L.). This knowledge can, in many ways, lead to optimal forest management, constituting a valuable tool in broader management and forestry contexts and offering a flexible solution, such as the estimation of the stem wood volume using easily obtained variables in the field, saving effort, and the prediction of the stem wood volume of trees that are no longer in the forest.

The suggested volume estimation system reduces the total stem volume estimation by 0.382 m^3 (58.4%), as compared to the current methodology used in this forest area. The proposed strategy is hoped to contribute to the rational and sustainable management of the study forest and to the conservation of the mountain forest ecosystem. However, this system should be further tested across different Scots pine (*Pinus sylvestris* L.) forests to ensure its safe use for stem volume estimation and prediction.

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